Multiple Objective LP (MOLP)
For Scheduling Repetitive Projects

by

Mazen Abdulghani Al-Sinan

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

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In Partial Fulfillment of the
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This thesis, written by **MAZEN ABDULGHANI AL-SINAN**, under the direction of his Thesis Committee, and approved by all the members, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirements for the Degree of **MASTER OF SCIENCE IN CONSTRUCTION ENGINEERING AND MANAGEMENT.**

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كلية الدراسات العليا
COLLEGE OF GRADUATE STUDIES

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Dedication

To my mother for her continuous moral support and to my father for everything.

ACKNOWLEDGEMENT

Acknowledgement is due to King Fahd University of Petroleum & Minerals for support of this research.

I would like to express my thanks, deep appreciation, and gratitude to Dr. Kamal M. Al-Harbi (Thesis Committee Chairman) for his great guidance and help. I would also like to express my thanks to the other members of my Thesis Committee, namely, Dr. Sadi Assaf and Dr. Shokri Selim for their comments, suggestions and sparing their valuable time.

Sincere thanks to Mr. Ram for typing the thesis. Finally, thanks to all those who helped me in completing this research.

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خلاصة الرسالة

اسم الطالب الكامل : مازن عبد الغني السنان

عنوان الدراســة : جدولة المشاريع المتكرره بإستخدام البرمجة الخطية المتعددة الدوال

التخــصــص : هندسة واداره التشيد

تاريخ الشهـاده : ٢٣ / ١١ / ١٩٩٢م

يمكننا جدوله المشاريع الانشائيه المتكرره بإتقان عن طريق استخدام طرق البرمجة الخطيه . وفي هذه الدراسة تم تطوير نموذج متعدد الاهداف لجدوله المشاريع المتكررة ويتكون النموذج من هدفين متعارضين .الهدف الاول عباره عن داله تقلل من تكلفه المشروع والهدف الاخر عباره عن داله تقلل من مده المشروع . والنموذج الذي طور عبر هذه الدراسه هو شامل في طبيعته وقادر على حل بعض من قصور النموذج ذو الهدف الواحد . والنتائج عند استخدام هذا النموذج هو عباره عن مجموعه تمثل كل المدد الممكنه للمشروع والتكلفه الإضافية المرتبطة بكل مده . وقد استخدم مثال لتوضيح هذا النموذج وكذلك استخدم برنامج قادر على حل المسائل الخطيه ذات الدوال المتعدده لحل المثال التوضيحي ، وأخيرا تم تقييم النموذج ومقارنته بالطرق التقليديه لجدواله المشاريع .

درجة الماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران ، المملكة العربية السعودية

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THESIS ABSTRACT

FULL NAME OF STUDENT	:	MAZEN ABDUL GHANI AL-SINAN
TITLE OF STUDY	:	MULTIPLE OBJECTIVE LP FOR SCHEDULING REPETITIVE PROJECTS
MAJOR FIELD	:	CONSTRUCTION ENGINEERING AND MANAGEMENT
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Construction of repetitive projects can be scheduled efficiently by using the Linear Programming Technique. In this study, the Multiple Objective model was developed to schedule repetitive projects. The model has two conflicting objective functions. One objective function is to minimize the project cost and another to minimize the project duration. The developed model is general in its nature and it overcomes the drawbacks that a single objective model has. The output of running this model is a set of all feasible durations and the associated additional minimization cost. The model is illustrated by a project example. The project example is implemented using a computer coded algorithm which is capable of solving Multiple Objective functions problems. Finally, the model was evaluated and compared with the Traditional Network Technique (CPM).

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

CONSTRUCTION ENGINEERING AND
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CHAPTER I

INTRODUCTION

1.1 General

Certain construction projects such as pipelines, highways, high-rise buildings, bridges, etc., have repetitive construction activities and they are called repetitive projects. The construction of these projects can be divided into stages where the resources performing the work flow from one stage to the next, provided that each stage consists of more or less the same activities.

Repetitive projects can be modeled (scheduled) optimally by linear programming (LP) techniques. These techniques are able to maintain work continuity which helps in maximizing learning curve effects. In other words, LP techniques increase the productivity and minimize the duration of the project while considering the cost.

The usual goal of a contractor is completing the project at minimum possible cost and time leading to minimizing project direct and indirect costs, i.e. minimizing project total cost. An individual activity cost tends to increase with decreasing activity duration up to a certain duration, called crashed duration (see Figure 1). The key element to control the cost of a project is to allocate the proper quantity of resources which are capable of maintaining both the cost and the duration of the project.

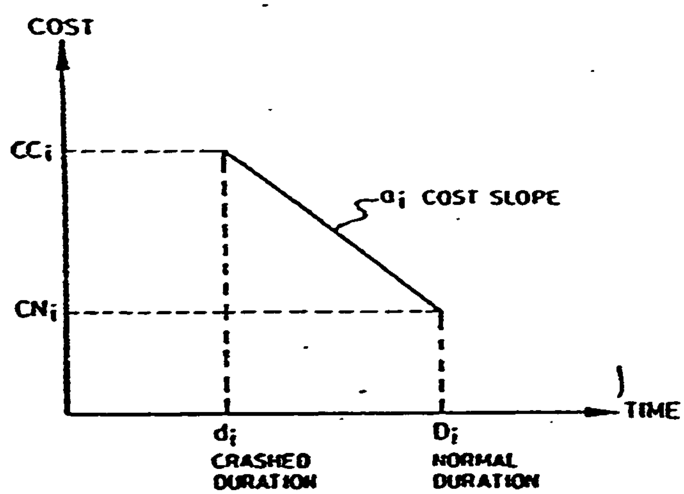


Fig. 1.1: Time Cost Curve (Reda, 1990).

1.2 Statement of the Problem

The single objective models for scheduling repetitive projects have some drawbacks and limitations. First, they have a single objective function for minimizing project direct cost. Since company and project indirect costs (overheads) are tied to the project duration, these models do not guarantee the minimization of the project total cost. As such, another objective function for minimizing project duration is needed, leading to the requirement of a multiple objective linear programming model (MOLP). Second, certain repetitive projects do not fit into the assumption that a single activity has the same duration in all stages. For example, the durations of an excavation activity is usually different from one stage to another of a pipeline project.

This study is intended to model repetitive projects through applying multiple objective linear programming technique. The developed model is capable of satisfying all constraints that a single objective model satisfies. Moreover, the model guarantees minimization of the project total cost.

Finally, the model does not assume constant duration for each activity in all stages. In other words, it is a general approach by which a single objective model can be considered a special case.

This general model is implemented on an illustrative project example to evaluate it and to compare it with single objective linear programming models and the traditional CPM approach.

1.3 Objective of the Study

The objective of the study will be to develop a model for scheduling repetitive projects. The model will minimize simultaneously both the project duration and the project cost. It will be based on multiple objective linear programming technique.

The model will satisfy the following constraints:

1. Minimize project total cost and duration.
2. Maintain constant production rate for each activity's crew.
3. Consider an activity that has different durations at different stages.
4. Allow for a stage buffer between concurrent activities.

The model will be developed mathematically and then it will be examined using a software package to evaluate the reliability of the model by using an illustrative example.

1.4 Scope

This study will be limited to modeling repetitive projects using multiple objective linear programming technique. Implementation of the model will require use of a computer package for solving MOLP problems.

1.5 Organization

This study will be organized into six chapters. A comprehensive literature review related to scheduling repetitive projects will be presented in Chapter II. Chapter II will include also an overview of repetitive projects, repetitive project model (RPM), and the drawbacks of RPM. Chapter III will present the proposed model and its development. It will also include a background related to multiple objective linear programming. Chapter IV will illustrate the model by implementing the model on an example to evaluate the validity and the reliability of the model. A computer software will be used in implementing the model on the project example. Chapter V will discuss the model and evaluate it. It will also compare the proposed model with the present technique for scheduling repetitive projects. The recommendations and conclusions of this study will be presented in Chapter VI.

CHAPTER II

LITERATURE REVIEW

2.1 Repetitive Projects

Repetitive projects are those projects that involve a long series of repetitive operations. Repetitive projects involve a sequence of i = 1,2,....P activities. Within each activity, work is performed sequentially from one stage to another over n = 1,2,....N stages. Where i is activity number, P is the last activity in each stage, n is the stage number and N is the last stage in the project. However, each activity is not necessarily included at every stage. The word "stage" is used as a reference to levels in a high-rise building, sections in pipeline construction or highways pavement, etc. (Clough 1962; Russell and Casetlon 1988; and Reda 1990).

For example, a pipeline project might include layout and clearing, followed in order by excavation, stringing of pipe, pipe laying, testing and backfill. The basic plan for this is shown in Figure 2.1 with excavation and pipe stringing being concurrent operations. However, no pipeline contractor would proceed in such a step-by-step fashion unless the pipelines were very short in length or some very limiting conditions applied. Rather, location and clearing work would get well under way. Excavation and pipe stringing would then start and pipe laying would proceed fairly closely behind. Pressure testing of the pipe is done in sections followed by backfilling. After the project gets "strung out" along the right-of-way, all of these operations will be moving ahead simultaneously, one stage following closely upon the heels of the next" (Clough 1972).

In the above example, one way to perform the work is to divide the pipeline into sections. Each section might be one mile or 100 feet, depending on the length of the pipeline, terrain and other job conditions (Clough 1972). The criteria for dividing any repetitive project varies from one project to another and it depends on the type of the project (pipeline, highway, high rise building) and the condition of each project.

Once the project has been divided into repeating sections "stages", a detailed schedule and plan for each stage can be developed. Overall schedule and plan can be now accomplished by linking these stages. Figure 2.2 shows the CPM network for the whole project after dividing it into stages. This method will enable the user to balance his crew and maintain work continuity.

2.2 Literature Review

The concept of scheduling time began a long time ago, but it is difficult to determine when it exactly began (Harris 1978). Man has realized the importance of time since he was living in a cave and the sunrise and sunset were the clock that he used. By time man has developed and with his development the means of measuring the time have developed and varied.

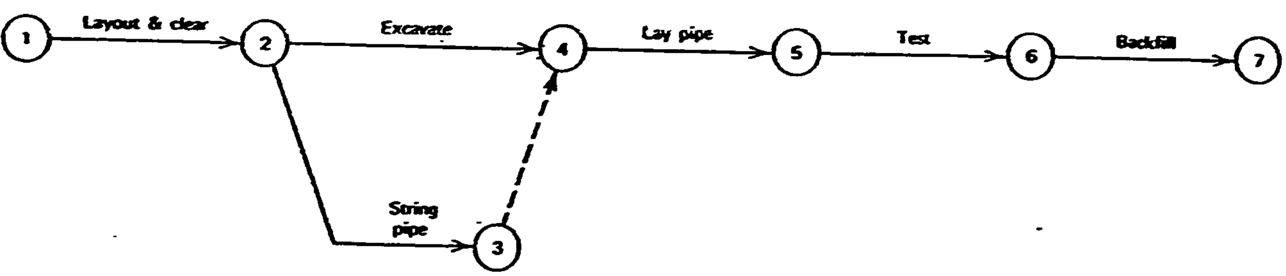


Fig. 2.1: Basic Pipeline Plan (Clough, 1972)

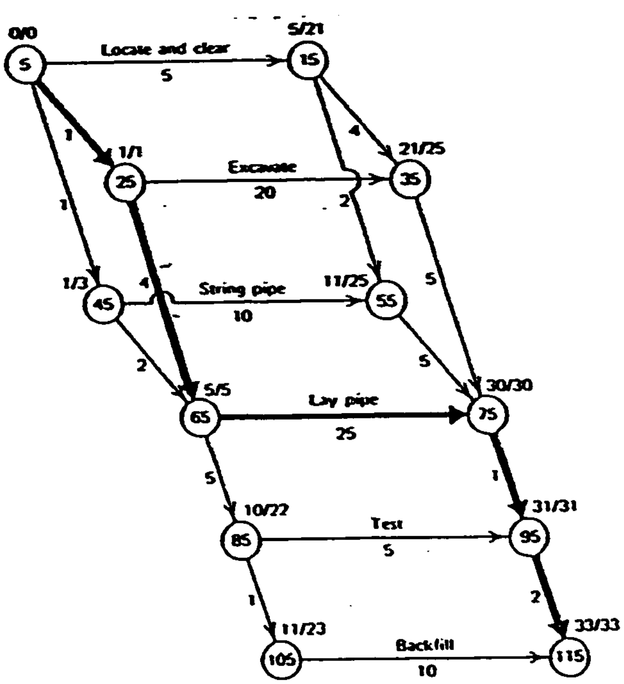


Fig. 2.2: Pipeline Project, Summary Diagram (Clough, 1972)

In the middle of the nineteenth century, industrial shops introduced the concept of scheduling time where the emphasis was upon maximizing the utilization time of the machinery (Harris 1978). As the shops became more complicated, the need for a more rapid and practical method of schedule introduced the graphical representation (Harris 1978).

The first scientific consideration of scheduling time was the graphical representation (Bar Chart) of work versus time which was developed by Henry L. Gantt and Fredrick W. Taylor during World War I (Harris 1978, McGough 1982). The Bar Chart is one of the most direct and easy methods to be understood when it is used for expressing project plans. A primary disadvantage of the Bar Chart is its failure to detail interrelationship of the activity. Because of this, the effect of change in activity sequencing and/or duration cannot be readily determined. (Harris 1978, McGough 1982, Chrzanowski and Johnston 1986). Figure 2.3 shows

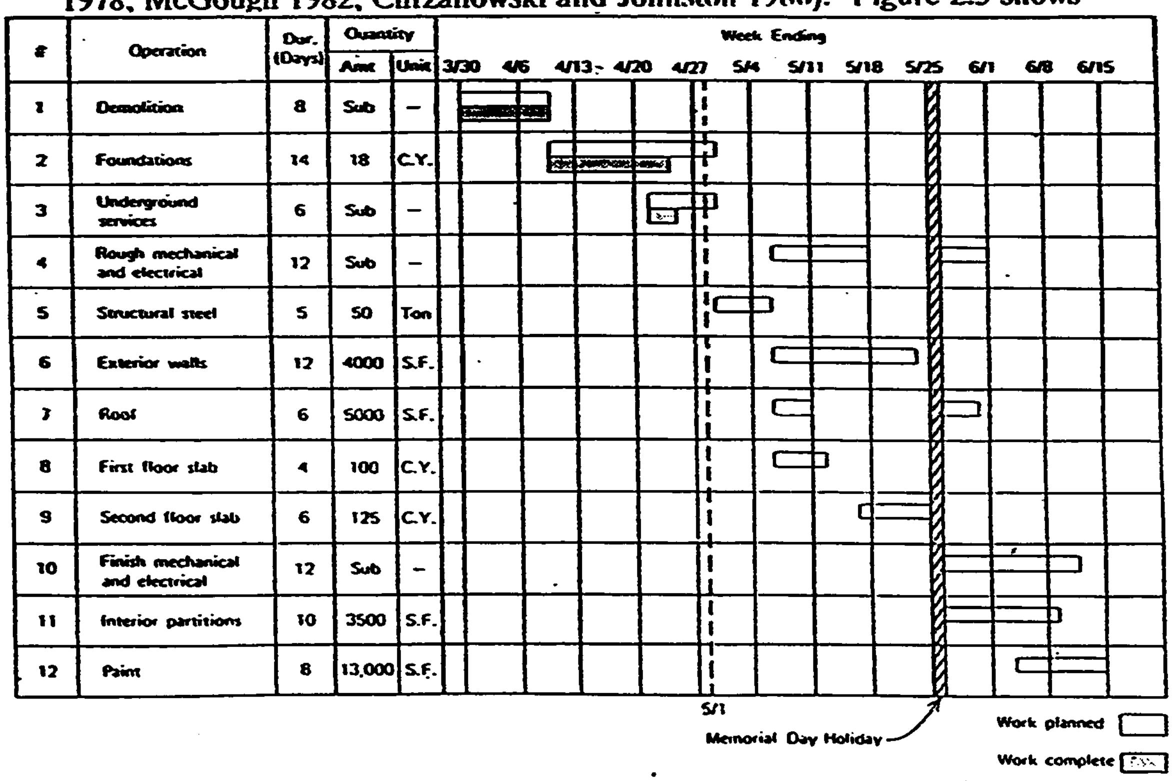
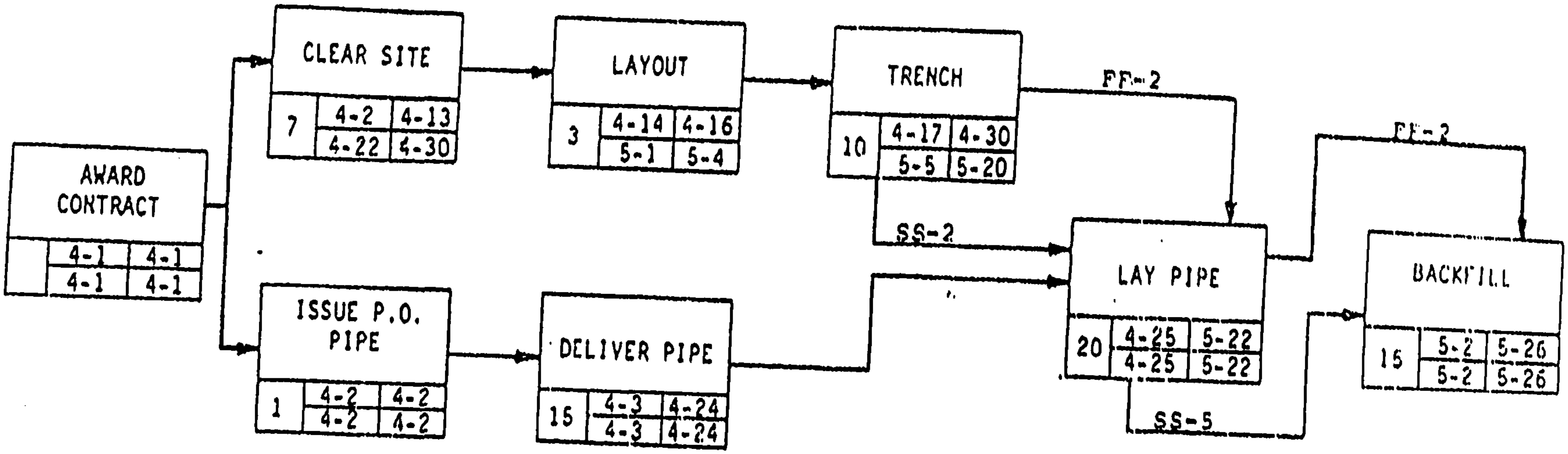


Fig. 2.3: Graphical Representation - Bar Chart (Harris, 1978)

2.2.1 The Traditional Network Approach

The Critical Path Method (CPM) was developed during the years 1957 and 1958 by Du Pont who was working for Sperry Rand Corporation to overcome the Bar Chart's failure to illustrate activity interrelationships and other weaknesses of the Bar Chart (Harris 1978, McGough 1982, Chrzanowski and Johnston 1986). CPM clearly illustrates the logical sequence of activities by representing the activities on connected nodes. The project duration and other information can be listed on a CPM diagram. The main disadvantage of the CPM is its difficulty to be understood and followed in field specially for a project with large number of activities. This problem is more obvious in repetitive projects, since the same activities are repeated throughout a project's duration (Chrzanowzki and Johnston 1986). The CPM schedule can be characterized with the repetition of information. Figure 2.4 shows an example of CPM schedule.



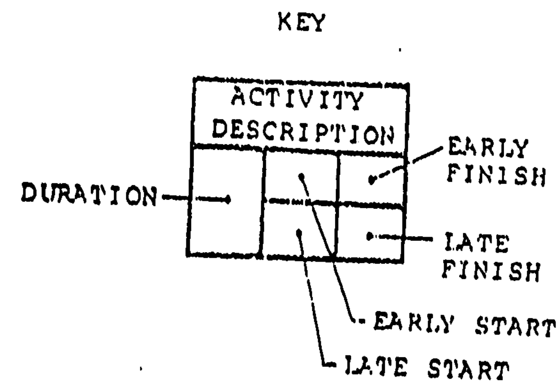


Fig. 2.4: CPM Network (McGough, 1982).

The Program Evaluation Research Task (PERT) was developed by the Navy Special Projects office as an integrated management technique for use in the Polaris Missile Program in 1957 (Harris 1978, McGough 1982). Both CPM and PERT use the network as the graphical model but the difference between them is that PERT assumes that the duration time of every activity has large variation while CPM assumes that the duration of each activity can be established with reasonable accuracy.

Network-based systems show the interrelationships and dependencies that control the project and have many advantages over the Bar Chart including the following (McGough 1982).

1. The overall plan and the ways in which the activities depend on or influence others can be seen.

2. The task of assessing the plan soundness and preventing unrealistic planning is facilitated.

3. Resource and time constraints can be included in the plan.

4. By dividing the project into smaller activities, more accurate estimation of duration can be made, resulting in increased accuracy of target dates.

5. Cost considerations can be included in the schedule.

The development of lead/lag techniques has improved representation of overlapping activities such as activities in linear projects and thus network analysis techniques were introduced (Johnston 1981). "The use of network analysis for construction scheduling has gradually increased, but not at the pace anticipated when it was first introduced in construction . The effort required to develop and update complex networks has discouraged many users" (Johnston 1981).

2.2.2 The Graphical Approach

Representation of linear activities is possible in a network, but the additional complexity has discouraged some in use of the method or interpreting the schedule (Johnston 1981). As a result, users preferred the simplicity of the Bar Chart regardless of its limitation in representing a project. An alternate approach for representing linear construction projects was needed. One method to represent linear construction projects was the graphical method which is used heavily in the Middle East (Johnston 1981).

The literature reveals that line-of-balance technique (LOB) was an early attempt to represent linear construction projects graphically. The LOB technique is a scheduling method developed by the U.S. Navy in the early 1950s. The LOB concept was first applied to industrial manufacturing production control, where the objective was to attain or evaluate a production line flow rate of finished products (Johnston 1981). O'Brien has summarized the LOB technique in relation to scheduling of manufacturing processes. Three diagrams are used in the LOB technique. An objective diagram shows the planned or actual (or both) number of units produced versus time. A progress diagram shows any particular date of interest during the production process. The third diagram is the progress diagram which shows the number of units for which each of the sub-assembly operations has been completed. (Johnston 1981) Figure 2.5 shows an example of the LOB schedule.

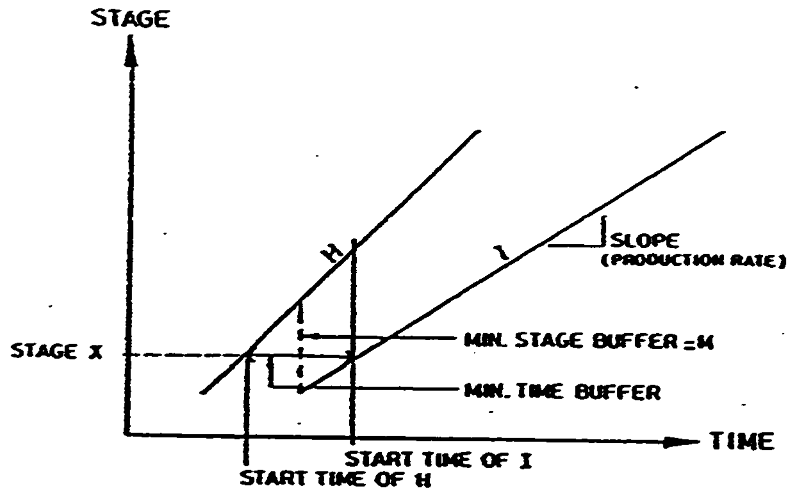


Fig. 2.5: Typical Flowline Curves of Repetitive Activities (Reda, 1990)

"Several practitioners have applied the LOB method in the construction industry as a scheduling technique. Lumsden (1968) modified the basic LOB technique and applied it to house construction scheduling. He explains in detail the principle of the method, its representation and the analysis" (Alsarraj 1990).

Khisty (1970) applied the LOB technique to construction in the classical sense of the manufacturing problem, given as examples the training of a large number of supervisors, production and supply of precast concrete beams, and improvements and repairs of a harbor. Carr and Meyer (1974) introduced the LOB technique in its present form. Carr and Meyer used the same method developed by Khisty (1970) to find the line-of-balance quantities at any time in the project life. The literature review reveals that the line-of-balance method was implemented mainly on repetitive projects.

Arditi and Albulak (1987) observed that LOB scheduling had some shortcomings. For example, there are major problems in the presentation of the LOB information. Unless colored graphics are used, it is difficult to differentiate between overlapping activities that have equal rates of production. Finally, the success of an LOB schedule depends a great deal on the setting of the rates of production of the many activities, which in turn depend on estimate of activity manhour requirements and crew size. The LOB schedule is extremely sensitive to errors and in these estimates that are magnified with increasing repetition.

Peer (1974) and Peer and Selinger (1972) developed a method called the Construction Planning Technique (CPT). CPT is a linear scheduling method, although it has been applied to projects having discrete activities as well. The efforts were to put CPT on mathematical form. Later Peer (1974) presented a discussion of the utilization of CPT and its advantages in comparison to network analysis method (Johnston 1981).

Another graphical approach was Vertical Production Method (VPM) which was introduced by O'Brien (1975). O'Brien concluded that some non-repetitive portions of high rise building projects such as foundations and substructure, can best be scheduled by network method. On the other hand, repetitive work on the typical floors is better scheduled using floor level versus time graph. O'Brien's viewpoint developed into VPM which is basically a linear scheduling method (Johnston 1981).

Halpin and Woodhead (1976) introduced a line of balance technique in relationship to high-rise building construction . The technique that they developed was a refinement of the Bar Chart which facilitated crew balancing but did not include a diagram similar to the Linear Scheduling Method (LSM) diagram.

2.2.3 Optimization Approach for Scheduling Repetitive Projects

The literature review does not indicate the exact origins of the linear scheduling method (LSM). There may actually have been multiple origins, possibly in different countries. Even the name used to designate the method has varied in the literature. The development of the LOM and the contributions of the above mentioned made it possible to develop the LSM. The LSM technique was implemented mainly on repetitive projects under different names, but within the same principles. Spang and Zimmerman (1968) presented a LSM diagram showing time versus distance along a tunnel for each of the major activities. Gorman (1972) (Johnston 1981) suggested the use of a "time versus distance diagram" in tunnel and highway construction . Johnston (1981) presented the LSM for Highway Construction and concluded the following :

1. LSM provides more information concerning the planned method of construction than a Bar Chart.

2. In repetitive portions of projects, LSM more quickly conveys the nature of the problem than network methods.

3. Among the transportation-related projects that could be scheduled using LSM are highway construction, highway resurfacing and

maintenance, airport runway construction and resurfacing, tunnels, mass transit systems, pipelines and railroads.

4. Although the method is not new, it has been given very little exposure among highway contractors.

5. The LSM scheduling method can assist in organizing construction work and reducing construction time.

Stradal and Cacha (1982) compared the principal scheduling techniques (Network, Bar Chart, and time spacing diagrams). They concluded that the time space scheduling is useful for linear projects. They called the technique "time space scheduling technique" because it shows clearly the connections of activities, their duration times and section (spaces) where they take place in a given time.

Linear Scheduling Method (LSM) has led to introducing optimizations to scheduling techniques for repetitive projects. Literature review reveals that there was a great effort in the recent past in developing managerial control systems involving classical optimization techniques such as simulation, queuing theory, linear dynamic programming, etc.

Crandall and Woolery (1982) developed a stochastic scheduling model to control the overall planning process rather than monitoring the routine fabrication or day-to-day construction. The model was based on Monte Carlo simulation network, from which variable "milestone data" are obtained. The normal Monte Carlo process yields a distribution for the time required for project completion, activity criticality and list potential critical paths. Crandall and Woolery (1982) developed a method whereby milestone data can be created at selected node points within the project without the necessity to maintain an entire record of each possible outcome of these nodes.

Handa and Barcia (1986) used the optimal control theory, a new branch of optimization, to view the construction - production process as a dynamic system that evolves over time. Their work was based on discretizing the dynamics of the system of a hypothetical cut-and-fill job on a section of a highway (Araditi and Albulak 1986).

Russell and Caselton (1986) presented a two-state variable n-stage dynamic programming formulation of the linear scheduling problem. Their formulation was based on the conventional dynamic programming framework with the objective of minimizing the project duration. Their method accounts for several of the realities of repetitive construction, including generalized precedence relationships and the ability to treat a variety of work continuity constraints. Also, they presented a method to extract sensitivity information which permits the identification of new optimal alternative solutions. Their contributions were directed towards the goal of having linear scheduling become a practical computer-based tool for repetitive construction projects.

The N-stage dynamic programming model uses two state vector variables: one to represent a set of possible duration required to complete work for an activity at each of the stages, and the second to represent a set of interrupt durations between work performed and adjacent stages. The model attempts to minimize the overall project duration, but does not guarantee a minimum direct cost of the project, since it considers the duration of each crew independently of direct cost of each activity.

The linear programming model, named the Repetitive Project Model or RPM, uses a single-objective function to minimize the project direct cost for each feasible project duration while satisfying the following constraints (Reda 1990):

1. Maintain a constant production rate for each activity's crew.
2. Maintain a continuity of work for each activity's crew.
3. Allow for a time buffer between activities on the same stage.
4. Allow for a stage buffer between concurrent activities.
5. Specify a feasible project duration.

2.3 The Single-Objective LP Model

2.3.1 Linear Programming Models

The common feature which mathematical programming models have is that they all involve optimization. The quantity which is to be maximized or minimized is called an objective function.

LP models have the following features (Williams 1990):

1. There are linear expressions (the objective functions) to be maximized or minimized.

2. There is a set of constraints in the form of linear expressions which must not exceed (≤) some specified value. Linear programming constraints can also be of the form (≥) and '=' indicating that the value of certain linear expressions must not fall below a specified value or must exactly equal a specified value.

3. The set of coefficients on the right hand side of the constraints is generally known as the right-hand side column.

Linear programming deals with finding extreme values of linear functions when the variables are constrained by inequalities.

A linear programming (LP) problem can be in the following form:

Minimize or maximize $\sum_{j=1}^{n} C_j X_j$ (objective function)

Subject to:

$$\sum_{j=1}^{n} a_i X_j \leq b_i , \quad 1 \leq i \leq m$$

$$X_j \geq 0 \qquad 1 \leq i \leq n$$

A linear Programming model can be of a single objective function or with multiple objective functions. Solving the model a number of times with each objective in turn might suggest a satisfactory solution to the multiple objectives model.

Objectives and constraints can often be interchanged. In fact, once a model has been built, it is easy to convert an objective function to a constraint or vice versa.

2.3.2 RPM: The Repetitive Project Model

The repetitive project model (RPM) is an approach for modeling repetitive construction projects. It incorporates a network technique, a graphical technique, and an analytical technique.

The network technique used in RPM uses a node on network. The graphical technique is used in RPM Line of balance technique.

The analytical model used in RPM is the linear programming model. The objective of RPM is to minimize the project direct cost for each feasible project duration while satisfying the following constraints (Reda, 1990):

1. Maintain a constant production rate for each crew.
2. Maintain a continuity of work for each crew.
3. Allow for a time buffer between activities on the same stage.
4. Allow for a stage buffer between concurrent activities.
5. Specify a feasible project duration.

The assumptions used for the development of the mathematical formulation of RPM are the following (Reda, 1990):

1. An activity on the node network is used to represent a typical stage of the project.

2. For each activity on the typical network, an activity time cost curve is developed. It is applicable for the same activity at all stages.

3. A constant duration is set for the same activity at all stages to maintain a constant production rate. Thus, if an activity needs to be changed to meet a particular feasible project duration, then an equal change must be made to the activity at all stages.

4. For each activity in the typical network, a flow line is used to represent the activity in the project flow line graph.

5. No idle time is allowed for any crew; thus, once a crew starts working on an activity at the first stage, it will continue working with the same production rate until it finishes work on last stage.

6. Each activity has a time buffer at each stage between the start time of the activity and the start time of each preceding activity. This buffer time equals the duration of the preceding activity plus any necessary lead time.

7. Any two concurrent activities may have a stage buffer of a specific number of stages to meet practical and/or technological purposes. This stage buffer has to be identified by the planner for these activities. The RPM model finds, for each feasible project duration, the start time of each activity at the first and last stage, the duration of each activity (which is to be set equal at all the stages to maintain a constant production rate for the crew working on the activity), and finally the project minimum direct cost that can achieve this project feasible duration.

2.3.3 Mathematical Formulation for RPM

Objective function

Minimize $\sum_{i=1}^{P} N(a_i y_i) + \sum_{i=1}^{P} N(CN_i)$ for $i = 1,2,\ldots P$ (1)

Subject to the following constraints:

$TSI_i - TSI_h + y_h \geq D_h + LT_{hi}$ for $i = 1,2,\ldots P$

for h: h is predecessor of i .... (1)

$TSN_i - TSN_h + y_h + \geq D_h + LT_{hi}$ for $i = 1,2,\ldots P$;

for h: h is predecessor of i .... (2)

$TSN_i - TSI_i + (N-1)y_i = (N-1)D_i$ for $i = 1,2,\ldots P$ (3)

$y_i \leq D_i - di$ for $i = 1,2,\ldots P$ (4)

$TSI_j - TSI_i + My_i \geq MD_i$ for $i = 1,2,\ldots P$

for j: a minimum stage buffer of M stages between i activity and j activity is needed. (5)

$TSN_i - TSI_j + (N-M-1)y_j \leq (N-M-1)D_j$ for $i = 1,2,\ldots P$

for j: a minimum stage buffer of M stages between i activity and j activity is needed. (6)

$TSN_p - y_p \leq FPD - D_p$ (8)

All variables are ≥ 0.0.

where:

a_i = cost slope of activity i;

CC_i = crash cost of activity i;

CN_i = normal cost of activity i;

D_i = normal duration of activity i;

FPD = feasible project duration desired;

M = number of stages desired as stage buffer between two activities;

N = number of stages in project;

P = total number of activities in typical network;

$TS1_i$ = start time of activity i at first stage;

TSN_i = start time of activity i at last stage;

y_i = number of days activity i is shortened to meet project feasible duration desired at minimum project direct cost.

Objective Function

The objective function attempts to minimize the normal cost plus the crashed cost needed for each activity in order to complete the project within the project duration specified in the constraints. N is the number of project stages, a_i is the cost slope of activity i, y_i is the number of days activity i can be shortened, CN_i is the normal cost of activity i and P is the maximum number of activities (Reda, 1990).

The first term of the objective function which calculates the crashed cost, reflects the assumption that if an activity needs to be changed, then an equal change should be made for the activity at all stages to maintain the same production rate. The decision variable in the objective function, y_i, represents the number of days each activity needs to be shortened, and is determined such that the value of the objective function is minimal and the project duration as set in the constraints is met (Reda, 1990).

In the objective function, the term $\sum_{i=1}^{P} N\,(CN_i)$ is constant. Thus, the term has no effect on the minimization process and, therefore, can be omitted. Also, the number of stages for any project, N, is constant and, therefore, can be omitted and the objective function can be rewritten as (Reda, 1990)

$$\text{Minimize} \sum_{i=1}^{P} a_i\, y_i \qquad i = 1,2, \ldots\ldots\ldots P$$

Constraints 1 and 2

Constraints 1 and 2 are applicable for each activity in the network to maintain a specified time buffer between starting time of activity i and each activity h that precedes it for each stage of the project. In order to achieve this, the time buffer between the start of activity i and h at the first stage ($TSI_i - TSI_h$) and last stage ($TSN_i - TSN_h$) are constrained to be greater than or equal to the time needed. This buffer time is equal to the normal duration of h minus the number of days h is shortened plus the additional time, if any, or (Reda 1990)

$$(TSI_i - TSI_h) \geq D_h - y_n + LT_{hi}$$

and

$$(TSN_i - TSN_h) \geq D_h - y_n + LT_{hi}$$

The production rate is assumed to be constant for each activity; therefore, once the time buffer is constrained at the first stage, then the time buffer will be maintained throughout.

Constraint 3

Constraint 3 is applied for each activity in the typical network to maintain the continuity of work with no idle time allowed for its crew. This is achieved by constraining the time needed to finish the activity throughout the project to be exactly equal to the sum of the activity's duration at all stages (Reda 1990).

Constraint 4

This constraint is applicable for all activities in the typical network to limit the number of days the activity can be shortened (y_i) to be less than or equal to the activity normal duration (D_i) minus the activity crashed duration (d_i) (Reda 1990).

Constraints 5 and 6

These two constraints are applicable for any two concurrent activities i and h that need a minimum number of stages as a buffer, M, between the crews for practical or technological purposes. This is satisfied through constraining the time that the crew working on activity j starts at the first stage to be greater than or equal to the time the crew is working on activity i starts working on M + 1 stage, and constraining the time that the crew working on activity i starts at the last stage, N, to be less than or equal to the time the crew working on activity j starts working on the N-M stage (Reda 1990).

If the activities i and j have unequal durations, then activity i should be the activity with the shorter duration to allow the two activities to finish in shorter time (Reda 1990).

Constraint 7

This constraint is to specify the project finish data, where TSN_p is the starting time of the last activity in the network, P, at the last stage, and FPD is the desired project finish date.

2.3.4 RPM Drawbacks

RPM has two drawbacks. First, it has a single objective function for minimizing project direct cost. Since company and project indirect (overhead) costs are tied to the project duration, the model does not guarantee the minimization of the project total cost. As such, another objective function for minimizing project duration is needed, leading to the development of a multiple objective linear programming model (MOLP).

Second, certain repetitive projects do not fit into the assumption of constant duration of each acativity in all stages. A single activity may have different durations at different stages of the project, assuming constant production rates. For example, the duration of an excavation activity is usually different from one stage to another of a pipeline project. Excavating rock materials take usually more time than excavating sand materials. Sellinger (1980) illustrated explicitly this idea in his paper's example.

CHAPTER III

A MULTIPLE OBJECTIVE LP MODEL

3.1 Time-Cost Relationships

For most activities in any project, there is a direct relationship between time and cost. On the other hand, some activities seem to have a fixed cost regardless of the duration and these activities are treated as constant values.

The relationship between time and cost is a proportional relationship and it can be approximated by a linear (straight-line) relationship. Figure 3.1 shows this relationship. The point C represents the minimum time and its coordinates are TC (crashed duration) and CC (crashed cost) on the time axis and cost axis, respectively. The point N is the minimum cost point and its coordinates are CN and TN on the cost axis and time axis respectively. The dashed curve represents the actual relationship between C and N. From this figure, it is obvious that there is trade-off between time and cost. The point E represents the conventionally derived estimate of the activity duration and cost. In reality, the point E should lie on the dashed curve, but the assumption of linearity places it on the solid line with anticipated small error (Harris 1978).

The point E is determined by usual estimating procedures, while the normal and crash points are not easy to obtain. The linear relationship between the costs allow for determination of cost slope for the activity. The cost slope for an activity may be expressed mathematically as:

$$S = \frac{CC - CN}{TN - TC} = \frac{\Delta C}{\Delta T}$$

where S is the cost slope, ΔC is the total change in costs and ΔT is the total change in time. The slope has a negative sign, but it will be ignored and the absolute level will be used in the adjustment procedures (Harris 1978).

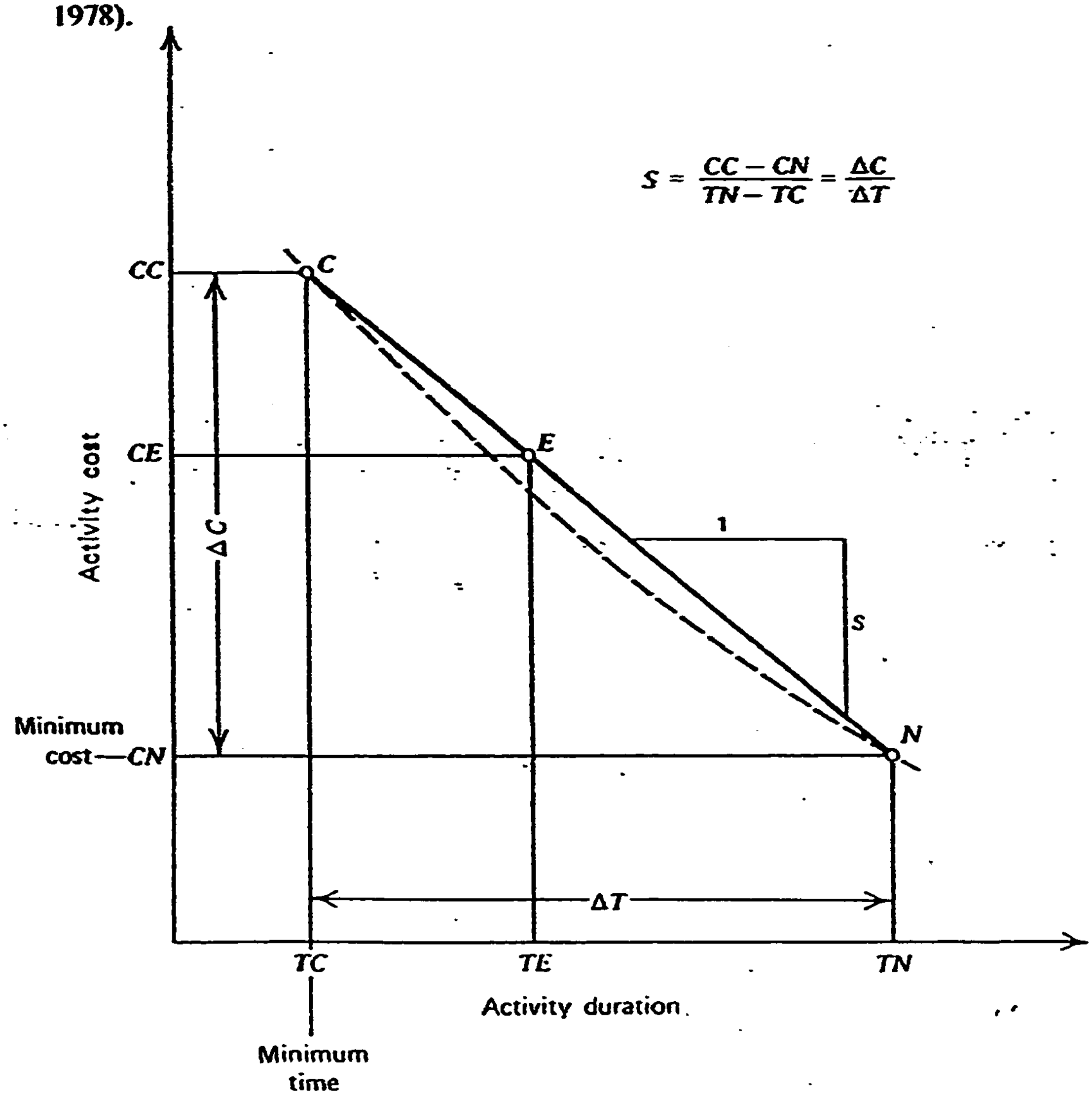


Fig. 3.1: Activity Time-Cost Relationships (Harris, 1978)

Total project cost can be divided into two types, direct costs and indirect cost. Direct cost are those that are assigned to the individual activities. Indirect costs are those that apply to the entire activities.

Most of the indirect costs are the general overhead that the construction firm must carry to do the business and this varies from one firm to another. The second portion of the indirect costs are those that apply to a particular project and this varies from one project to another. Indirect costs tend to increase with time and the rate of increase tends to vary linearly.

Direct costs for an activity tend to decrease with increasing activity duration. Project duration is calculated by summing activity duration along the critical path. Therefore, the direct project costs tend to decrease with an increase in project duration.

An activity can be performed at any time duration between its normal and crash points. Therefore, a wide range of possible combinations of schedule time and costs exist for every project time duration. "The combination of activity times and costs that gives the minimum value at every project duration results in the minimum direct project cost." (Harris, 1978). Completing the project at the minimum total cost is the objective for a contractor.

3.2 Multiple Objective Linear Programming (MOLP)

A good project scheduling approach attempts to optimize conflicting objectives which are utilizing limited resources. Time and cost can be a good example of conflicting objectives. Minimizing project duration requires increasing the cost and vice versa.

Multiple objective programming provides a method for evaluating constrained problems with conflicting objectives. Multiple objective programming develops a set of solutions called an efficient set, rather than the single solution determined by single objective models.

The multiple objective linear program (MOLP) can be expressed as follows:

$$\text{Minimize } C\,x$$
$$x \in F$$
$$F = \{\, x \in R^n \mid Ax - b \geq 0,\ x \geq 0 \,\}$$

where:

$C =$ q x n matrix of coefficients of q objective functions.

$x =$ n x 1 vector of decision variables.

$A =$ m x n matrix of constraint function coefficients.

$b =$ m x 1 vector of constraint right hand sides.

$R^n =$ set of all real n-dimensional vectors.

$C_i =$ 1 x n vector of the i-th row of C.

$X^* \in F$ is an efficient solution of a MOLP if and only if there is no $x \in F$ such that $C\,x \leq C\,x^*$ and $C_i\,x < C_i\,x^*$ for at least one i.

The solution is a set of points where each point is a feasible solution which is not dominated by any other feasible point. Points which are not in the set are dominated by one or more points in the efficient set and, therefore, are not valid combinations of variables that will optimize the stated objectives.

Many models which consider the relationship between time and cost in project scheduling have been developed.

Sweeney and Williams (1982) expressed the standard crashing model as follows:

$$\text{Minimize } Z = \sum_{\alpha=1}^{m} K_\alpha Y_\alpha \qquad \text{(Objective)}$$

Subject to:

$$X_j - X_i + Y_\alpha \geq T_\alpha \quad \text{for each activity } \alpha, \alpha = (i,j) \qquad (1)$$

$$Y_\alpha \leq M_\alpha \quad \text{for each activity } \alpha \qquad (2)$$

$$X_n \leq D \qquad (3)$$

$$X_i \geq 0 \quad \text{for all } i \qquad (4)$$

$$Y_\alpha \geq 0 \quad \text{for all } \alpha \qquad (5)$$

where:

X_i = Realization time of event i

Y_α = The amount of crashing for activity α

m = The number of activities.

n = The number of nodes in the network.

X_n = Terminal node.

T_α = Normal time for activity α.

M_α = Maximum amount for crashing activity α.

K_α = The cost per period required to crash activity α.

D = The due date of the project.

Constraint (1) preserves the precedence between activities in the project network. Constraint (2) prevents any activity from being crashed beyond the absolute minimum time required to complete the task. Constraint (3) assures the project will be completed on time, as X_n, the realization time of the terminal event, must be less than or equal to the project due date D. Constraints (4) and (5) are the standard non-negativity conditions.

The extension of the above model provides a multiple objective formulation as follows:

$$\text{Minimize } Z_1 = X_n$$

$$\text{Minimize } Z_2 = \sum_{\alpha=1}^{m} K_\alpha Y_\alpha$$

Subject to:

$$X_j - X_i + Y_\alpha \geq T_\alpha \quad \text{for all x}$$

$$Y_\alpha \leq M_\alpha$$

$$X_n \geq D$$

$$X_i \geq 0 \text{ for all i}$$

$$Y_\alpha \geq 0 \text{ for all } \alpha$$

The first objective, Z1, will minimize the project duration. The second objective, Z2, will simultaneously minimize the cost of crashing the project. These two objectives will give an efficient net for the project schedule.

Elmaghraby (1977) suggested an alternative format for the crashing model, stated as follows:

$$\text{Minimize } Z = \sum_{(i,j)\in A} C_{ij} = \sum_{(i,j)\in A} (b_{ij} - a_{ij} Y_{ij})$$

Subject to:

$$X_i - X_j + Y_{ij} \leq 0 \quad (i,j) \in A \tag{1}$$

$$Y_{ij} \leq \mu_{ij} \tag{2}$$

$$Y_{ij} \geq \alpha_{ij} \tag{3}$$

$$X_n \leq D \tag{4}$$

Where:

X_i = realization time of event i

Y_{ij} = duration of activity i,j

μ_{ij} = Upper bound on the duration of activity i,j

α_{ij} = Lower bound on the duration of activity i,j

$C_{ij} = b_{ij} - a_{ij} Y_{ij}$ = Cost of activity i,j as a function of the activity duration.

b_{ij} = Non-negative constant.

a_{ij} = Non-negative constant.

A = Set of all activities in the project.

n = Number of the terminal node.

D = Maximum project duration.

The objective function can be in the following form:

$$\text{Minimize } Z_1 = \sum_{(i,j)\in A} (b_{ij} - a_{ij} Y_{ij})$$

$$\text{Minimize } Z_1 = \sum_{(i,j)\in A} b_{ij} - \sum_{(i,j)\in A} a_{ij} Y_{ij}$$

$$\text{Minimize } Z_1 = \sum_{(i,j)\in A} a_{ij} Y_{ij} \quad \text{; since } b_{ij} \text{ is constant.}$$

Project duration can be minimized as Minimize $Z_2 = X_n$

The multiple objective approach provides flexible method to model the time/cost tradeoff problem. However, a single objective linear program can be solved much more quickly than a multiple objective linear program. A number of computer algorithms to solve MOLP model are available.

3.3 MOLP Model for Scheduling Repetitive Projects

The Multiple Object Linear Programming (MOLP) model for scheduling repetitive projects proposed in this study is a unified approach for modeling repetitive construction projects. It incorporates network technique, a graphical technique and an analytical technique.

The network technique used is precedence network technique where activities are presented on nodes and dependencies between activities are represented by arcs. The use of the network is to represent the activities and their dependencies for a typical stage of a repetitive project.

The graphical technique used in MOLP model uses line of balance technique (LOB). Line of balance (LOB) involves time cost curve as shown in Figure 3.2 and activity flow line as shown in Figure 3.3 (Reda 1990). The time cost curve represents the relation between the cost and duration. Also, it draws the crashed duration and crashed cost for each activity. The activity flow line curve represents the movement of an activity crew from one stage to another, throughout the project stages. The slope of each line represents the two production rate for each activity. The horizontal distance between the two lines at any stage represents the buffer time between the two activities at the stage. The vertical distance between the two lines represents the stage buffer between the two activities at a particular time (Reda 1990).

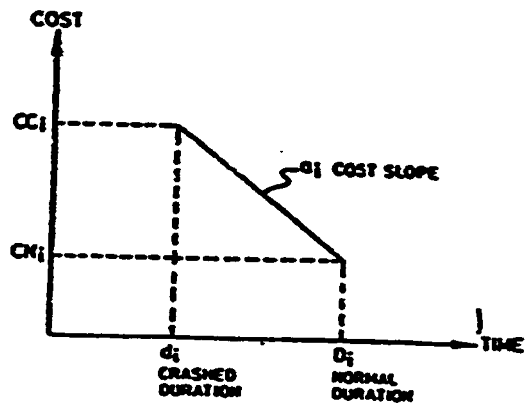


Fig. 3.2: Time Cost Curve (Reda 1990)

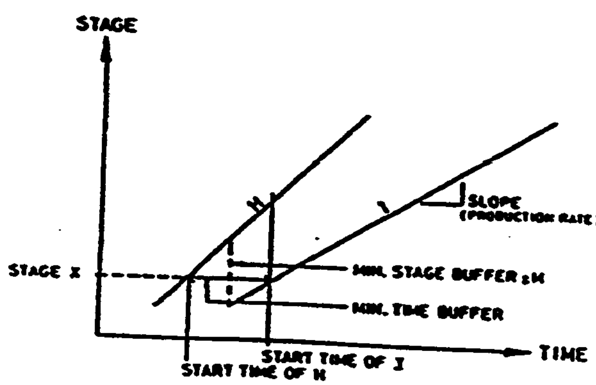


Fig. 3.3: Activity Flow Line.(Reda 1990)

The intersections of a horizontal line drawn at a particular stage with the flow lines give the starting time of activities at stage. The intersections of a vertical line drawn at any particular time the flow lines give the activities concurrently performed at different stages. Using a flow line for each activity in the typical network, a project flowline curve can be developed showing flow lines for all activities at all stages (Reda 1990).

The analytical technique used is a multiple objective linear programming (MOLP) model. The objective of the MOLP model is to minimize the project cost and duration while satisfying the following constraints:

1. Maintain a constant production rate for each activity's crew.
2. Maintain a continuity of work for each activity's crew, considering that an activity may have different durations at different stages of the project.
3. Allow for a time buffer between activities on the same stage.
4. Allow for a stage buffer between concurrent activities.

The mathematical formulation of MOLP model consists of two objective functions and 6 practical constraints. The following assumptions are used for the development of the mathematical formulation of MOLP model and were listed by (Reda 1990):

1. An activity on node network is used to represent a typical stage of the project.
2. For each activity on typical network, an activity time cost curve is developed.

3. The duration of each activity is set for each stage.
4. For each activity in the typical network, a flow line is used to represent the activity in the project flow line graph.
5. Rate of production may vary from one stage to another, thus crew's working on an activity at the first stage may have different production rate in other stages.
6. Each activity has a time buffer at each stage between the start time of the activity and the start time of each preceding activity. This buffer time equals the duration of the preceding activity plus any necessary lead time.
7. Any two concurrent activity may have a stage buffer of a specified number of stage to meet practical and/or technological purposes.

The MOLP model provides an effective method to analyze conflict objectives in project scheduling. It considers the relationship between the time (duration) and cost of a project schedule. The MOLP model generates set of efficient solutions (time and cost) which satisfy the constraints of the model.

3.4 Mathematical Formulation for MOLP Model

Objective functions:

$$\text{Min } Z_1 = \sum_{s=1}^{N} \sum_{i=1}^{P} a_{is} y_{is} + \sum_{s=1}^{N} \sum_{i=1}^{P} CN_{is} \qquad (1)$$

$$\text{Min } Z_2 = TS_{PN} + D_{PN} \quad Y_{PN} \qquad (2)$$

Subject to the following constraints:

$$TS_{is} - TS_{hs} + y_{hs} \geq D_{hs} + LT_{hi}$$

for i = 1,2,....p and h is predecessor of i (3)

$$TS_{i(s+1)} - TS_{is} + y_{is} \geq D_{is}$$

for i = 1,2,....p (4)

$$TS_{iN} - TS_{i1} + \sum_{s=1}^{N-1} y_{is} = \sum_{s=1}^{N-1} D_{is}$$

for i = 1,2,....p (5)

$$y_{is} \leq D_{is} - d_{is}$$

for i = 1,2,.....P and S = 1,2,....N (6)

$$TS_{js} - TS_{is} + \sum_{s=1}^{M} y_{is} \geq \sum_{s=1}^{M} D_{is}$$

for i = 1,2,...P and S = 1,2,....N (7)

for j: a minimum stage buffer of M stages between i and j is needed..

i and j are concurrent activities.

$$TS_{iN} - TS_{j1} + \sum_{s=1}^{N-M-1} y_{js} \leq \sum_{s=1}^{N-M-1} D_{js} \quad (8)$$

for j: a minimum stage buffer of M stage between i and j is needed.

i and j are concurrent activities.

All variables are $\geq$ 0.0.

where:

N = Number of stages in project;

a_{is} = Cost slope of activity i in the s^{th} stage;

y_{is} = Number of days activity i is shortened in the s^{th} stage;

CN_{is} = Normal cost of activity i in the s^{th} stage;

P = Number of activities in the stage;

TS_{is} = Starting time of activity i in the s^{th} stage;

TS_{PN} = Starting time of the last activity (p^{th}) in the last stage (N^{th})

D_{PN} = Normal duration of the last activity (p^{th}) in the last stage (N^{th})

Y_{PN} = Number of days the last activity (p^{th}) in the last stage (N^{th}) is shortened;

D_{is} = Normal duration of activity i in the s^{th} stage;

d_{is} = Crashed duration of activity i in the s^{th} stage;

LT_{hi} = Lead time between activity i and h.

the additional lead time, if any (Reda 1990).

Constraint 4 maintains the order for each activity between different stages. This constraint is needed, since each activity might have different durations at each stage.

CONSTRAINT 5

Constraint 5 maintains the continuity of crew work on each activity at each stage. It constraints the time needed to finish the activity throughout the project to be exactly equal to the sum of the activity's durations at all the stages.

CONSTRAINT 6

Constraint 6 limits the number of days each activity can be shortened at the s^{th} stage (y_{is}) to be less than or equal to the activity normal duration at the s^{th} stage (D_{is}) minus the activity crashed duration at the s^{th} stage (d_{is}).

CONSTRAINTS 7 AND 8

Constraints 7 and 8 are written for any two concurrent activities i and j that need a minimum number of stages as a buffer, M, between the crews for practical and/or technological purposes. It constraints the time that the crew working on activity j starts at the first stage to be greater than or equal to the time the crew working on activity i starts working on the M + 1 stage, and constraints the time that the crew working on activity i start at the stage, N, to be less than or equal to the time the crew working on activity j starts working on the N-M stage. It should be noted that if the two activities i and j have unequal durations, then activity i should be the activity with the shorter duration to allow the two activities to finish in shorter time (Reda 1990).

CHAPTER IV

ILLUSTRATIVE PROJECT EXAMPLE

4.1 Project Description

A project example follows to illustrate the MOLP mathematical formulation. The project example shown on Figure 4.1 was used by Reda (1990) who adopted it with modification from Clough and Sears (1979). This project example is modified to take into account the assumption of having different durations for some activities in different stages. This assumption was not considered by Reda (1990).

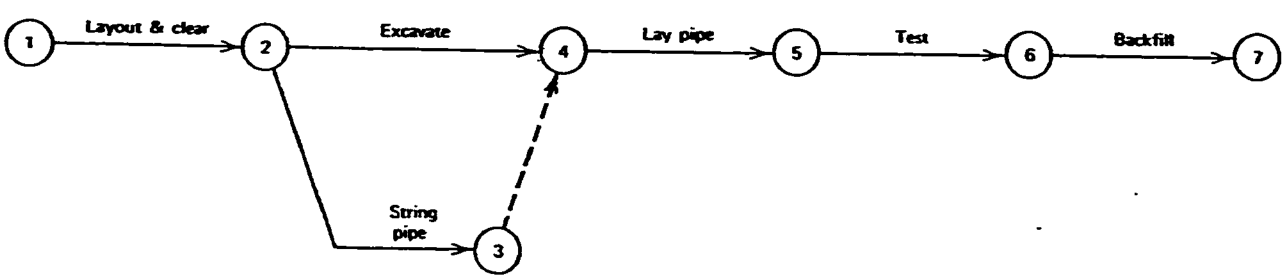


Fig. 4.1: Basic Project Example Plan (Clough, 1972).

The project example is the relocation of 5 miles (8 KM) of natural gas pipeline. The repeated segments of the pipeline relocation have been identified as:

Activity 1:	Locate and Clear	(LC)
Activity 2:	Excavate	(EX)
Activity 3:	String Pipe	(ST)
Activity 4:	Lay Pipe	(LA)
Activity 5:	Test	(TE)
Activity 6:	Backfill	(BF)

The project is divided into five stages and each stage represents 1 mile. Figure 4.2 shows an activity on node network for each stage of the project (Reda 1990).

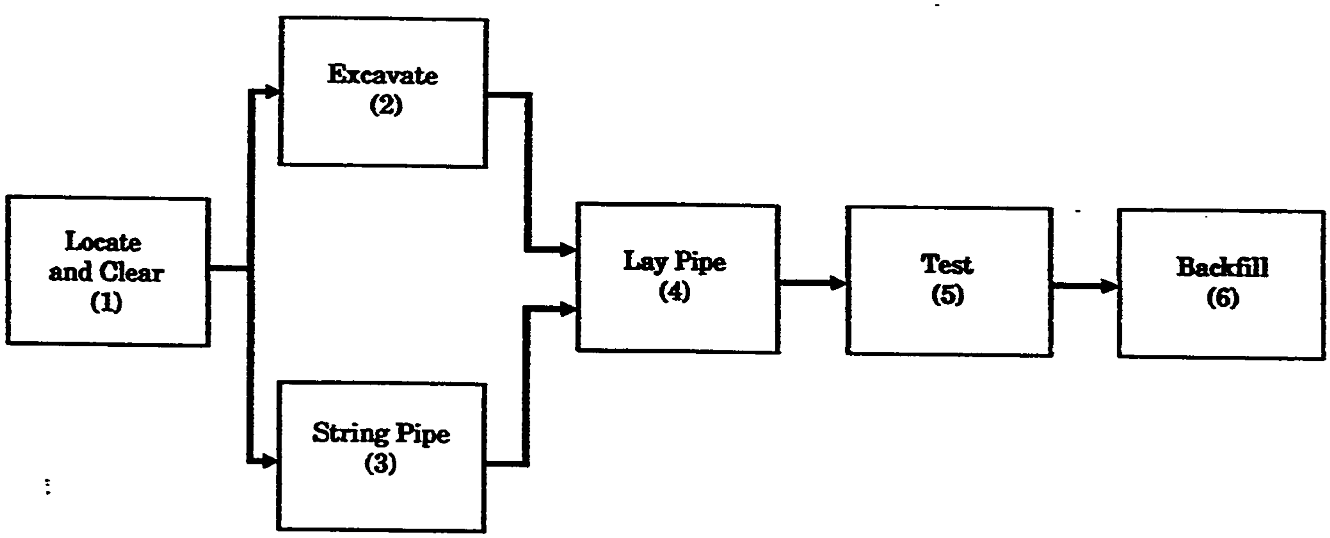


Figure 4.2: Typical Stage Network of Example Project.

Reda's basic example assumes that all activities have the same duration in all stages, shown in Table 4.1. A more general case is that some activities have different durations at different stages of the project, as shown in Table 2. Figures 4.3 through 4.13 show the time-cost curves for each activity at each stage of the project.

Table No.4.1 : Reda's Basic Example Data

Activity	Normal Duration (Days)	Crashed Duration (Days)	Cost Slope ($/day)
Locate and clear	1	1	-
Excavate	4	1	50
String pipe	2	1	50
Lay pipe	5	1	150
Test	1	1	-
Backfill	2	2	-

Table 4.2: Cost and Duration Data for Project Example.

Activity	Stages 1 and 3			Stage 4			Stages 2 and 5		
	Normal Duration	Crashed Duration	Cost Slope	Normal Duration	Crashed Duration	Cost Slope	Normal Duration	Crashed Duration	Cost Slope
Locate and clear	1	1	-	1	1	-	1	1	-
Excavate	4	1	50	6	2	100	3	1	50
Stringe Pipe	2	1	50	4	2	100	3	1	50
Lay Pipe	5	1	100	6	2	150	6	2	200
Test	2	2	-	2	1	100	2	2	-
Backfill	1	1	-	4	2	100	1	1	-

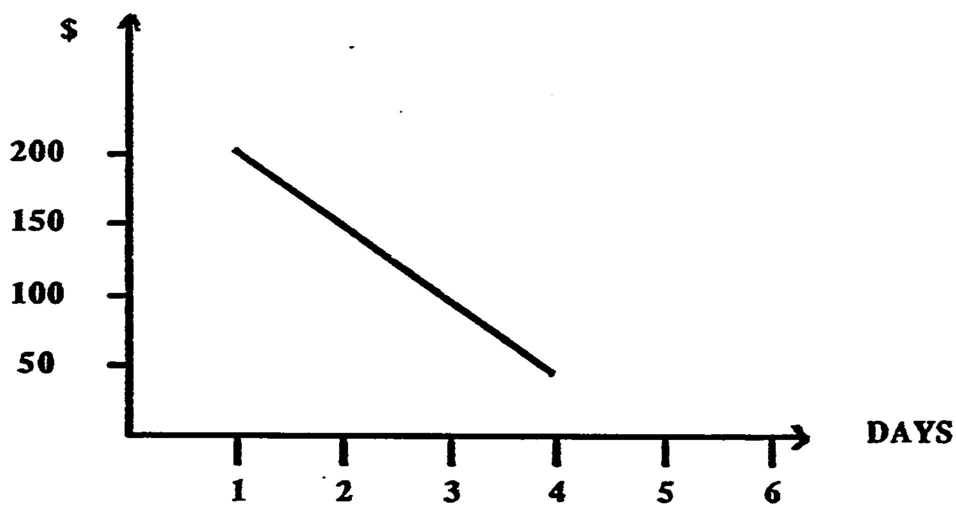


FIGURE 4 . 3

TIME - COST CURVE FOR EXCAVATION ACTIVITY

(STAGES 1 AND 3)

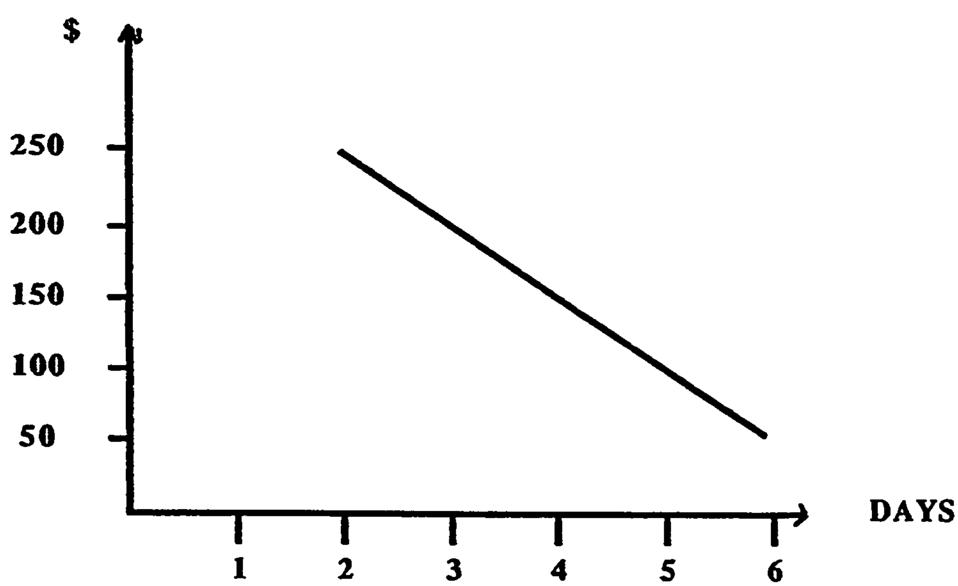


FIGURE 4 . 4

TIME - COST CURVE FOR EXCAVATION ACTIVITY

(STAGES 1 AND 3)

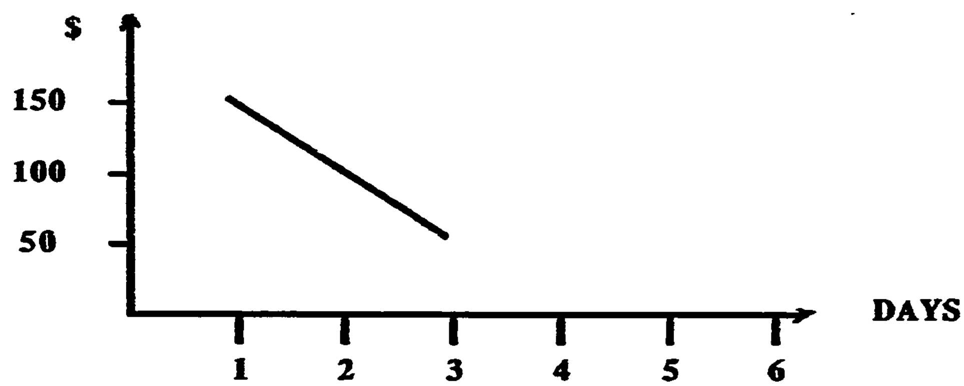


FIGURE 4 . 5

TIME - COST CURVE FOR EXCAVATION ACTIVITY

(STAGES 2 AND 5)

FIGURE 4 . 6

TIME - COST CURVE FOR STRING PIPE ACTIVITY

(STAGES 1 AND 3)

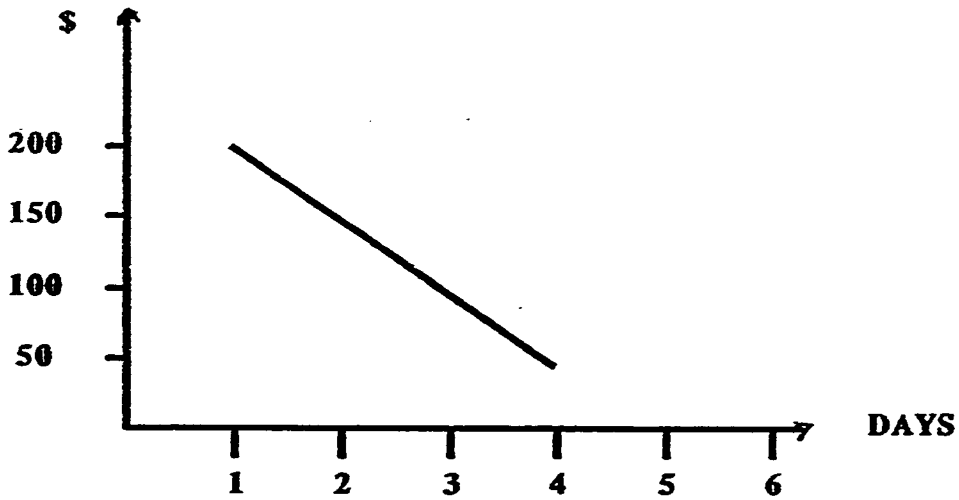


FIGURE 4 . 7

TIME - COST CURVE FOR STRING PIPE

(STAGES 4)

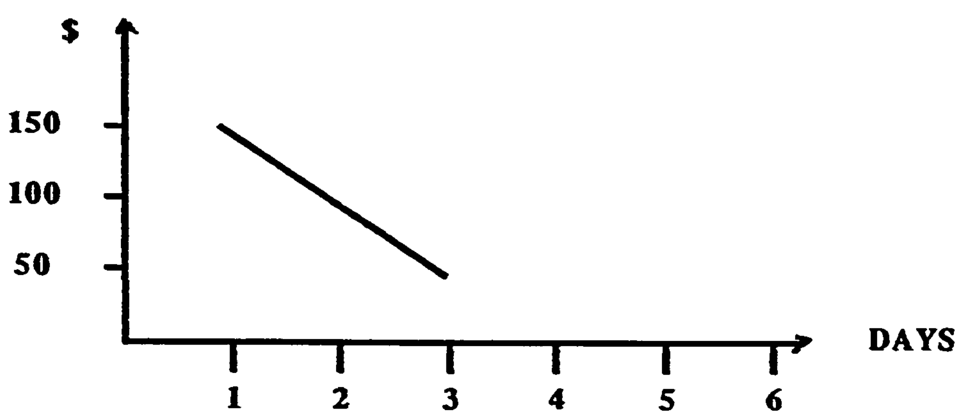


FIGURE 4 . 8

TIME - COST CURVE FOR STRING PIPE ACTIVITY

(STAGES 2 AND 5)

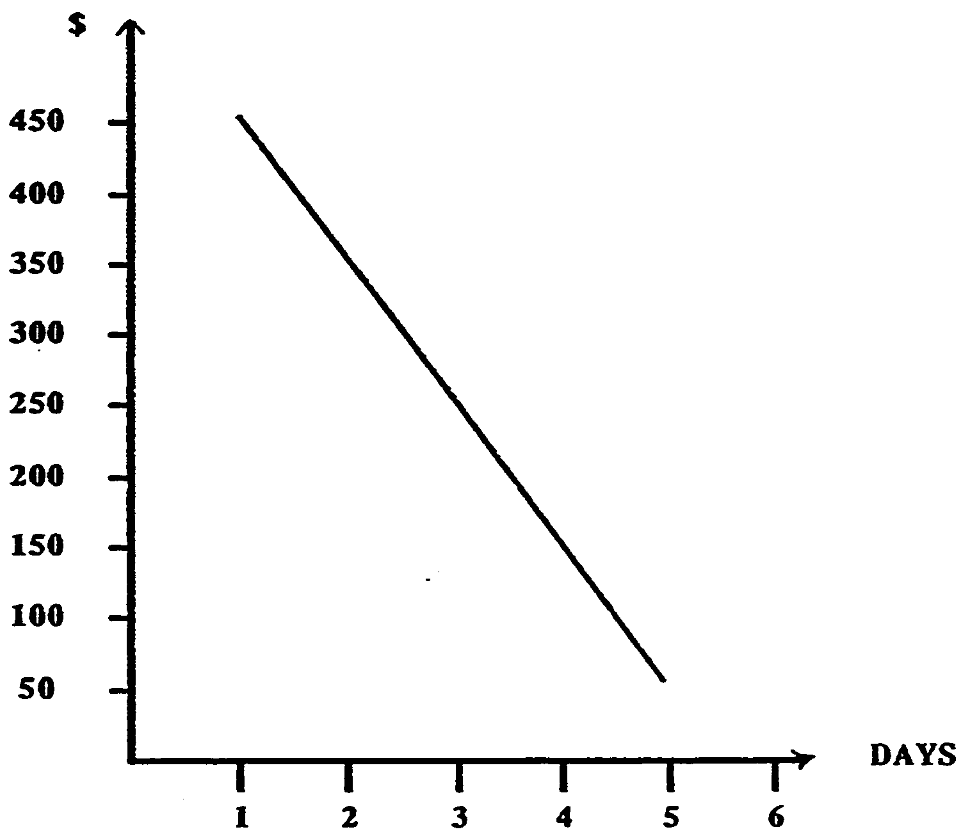


FIGURE 4 . 9

TIME - COST CURVE FOR LAY PIPE ACTIVITY

(STAGES 1 AND 3)

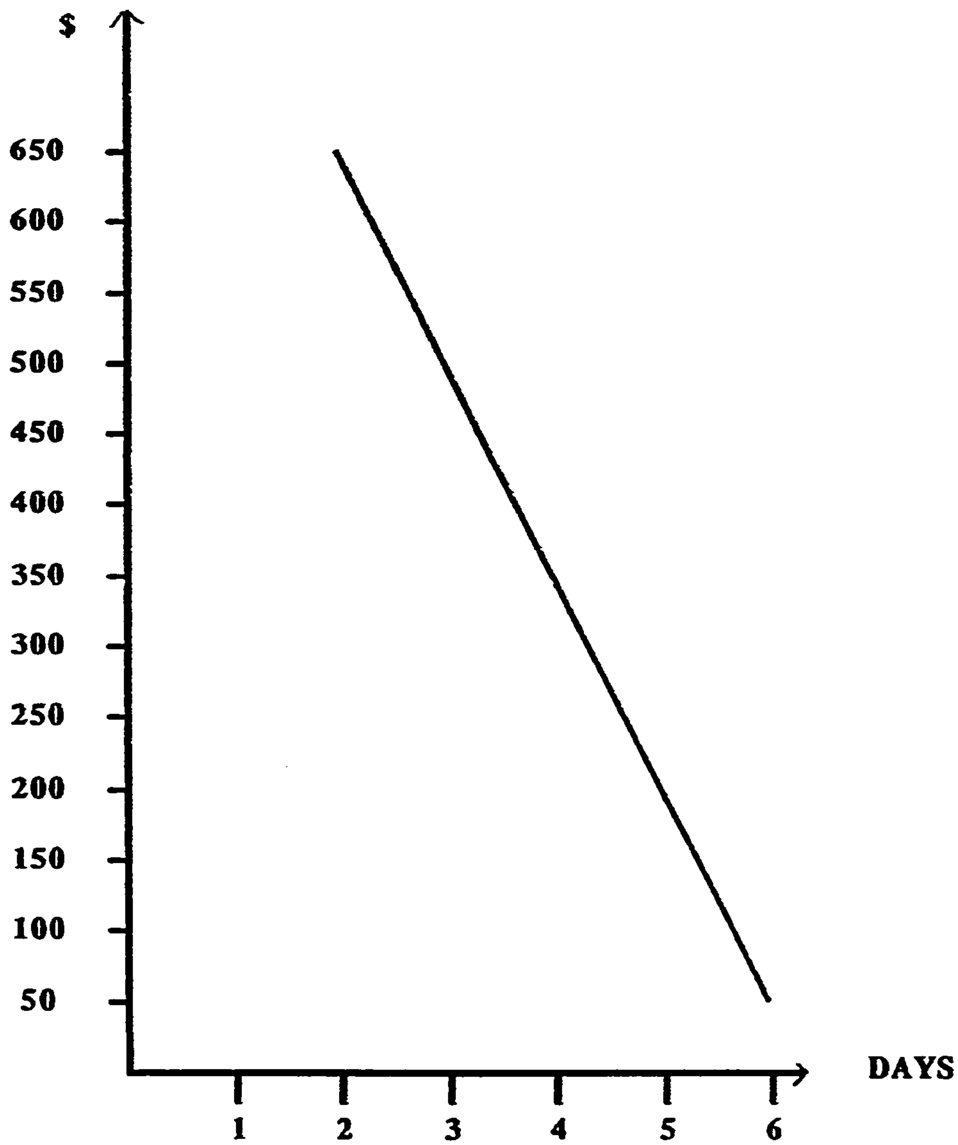


FIGURE 4 . 10

TIME - COST CURVE FOR LAY PIPE ACTIVITY

(STAGES 4)

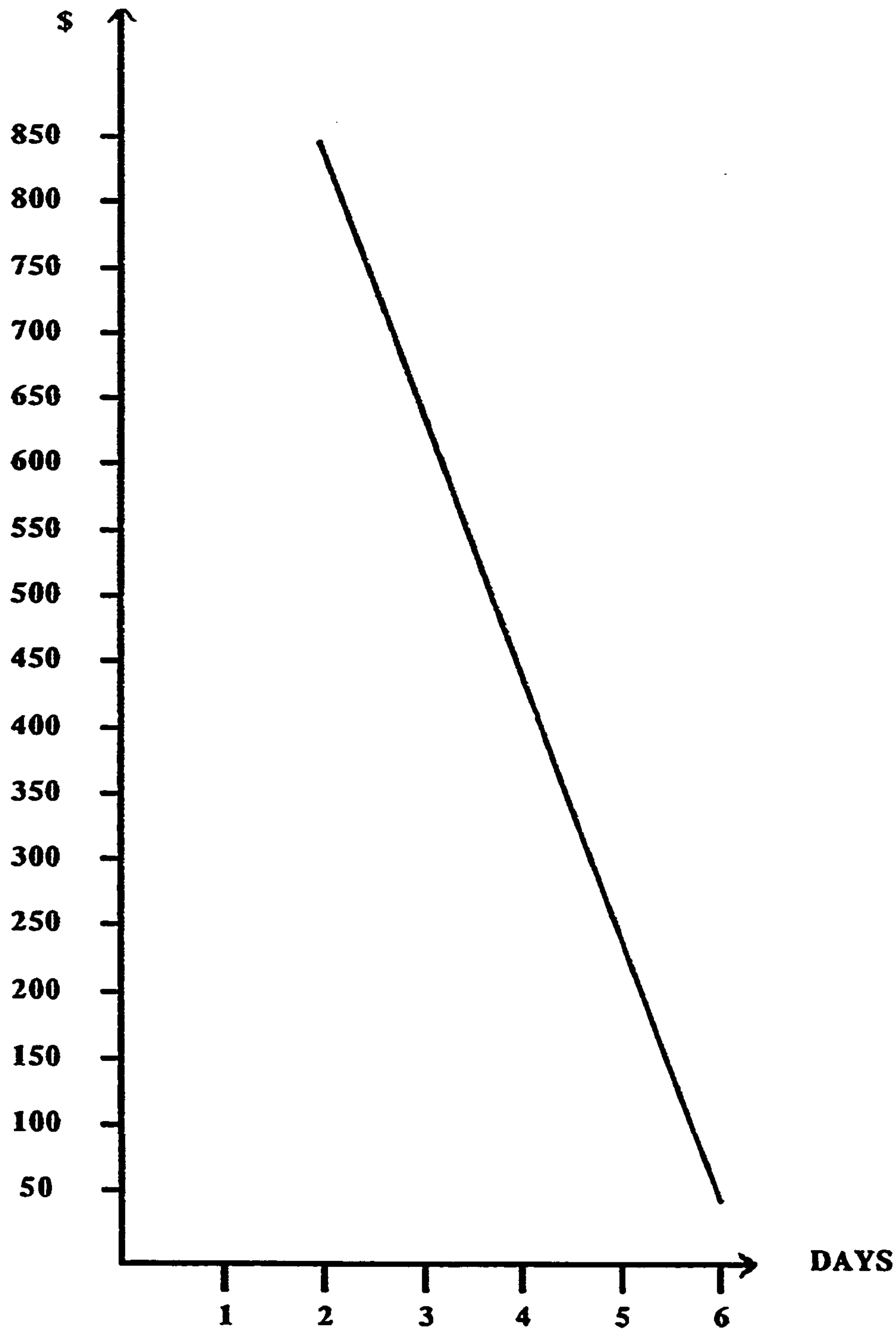


FIGURE 4 . 11

TIME - COST CURVE FOR LAY PIPE ACTIVITY

(STAGES 2 AND 5)

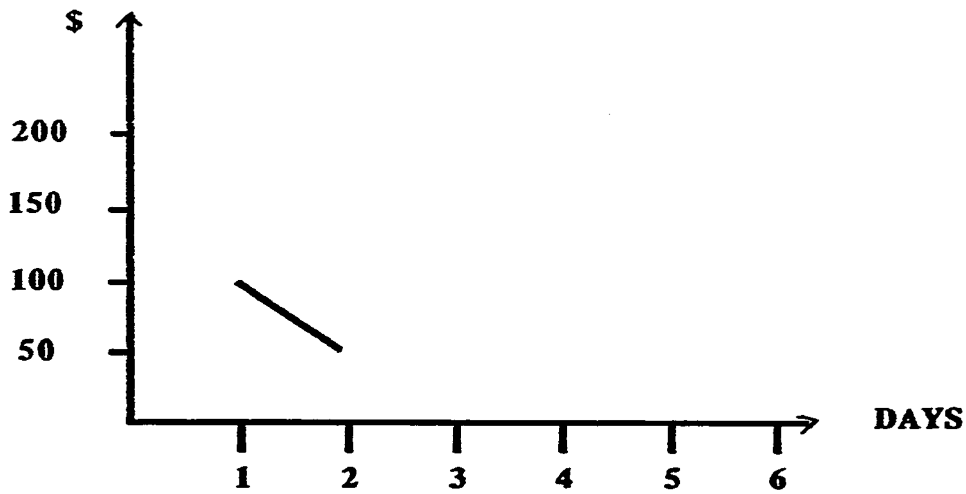


FIGURE 4 . 12

TIME - COST CURVE FOR TEST ACTIVITY

(STAGES 4)

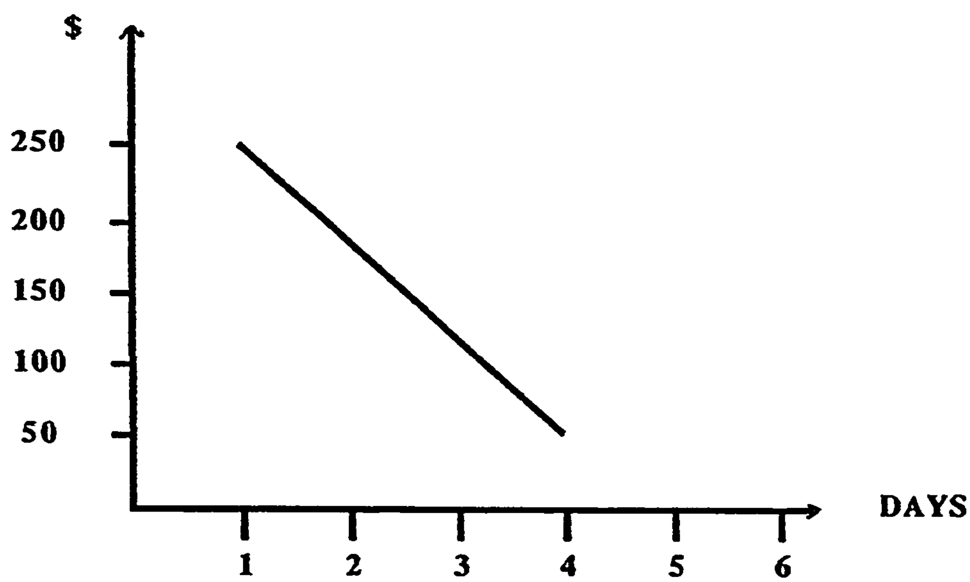


FIGURE 4 . 13

TIME - COST CURVE FOR BACK FILL ACTIVITY

(STAGES 4)

4.2 Mathematical Formulation for Project Example

OBJECTIVE FUNCTION

Min

$Z_1 = 50\,(y_{21} + y_{22} + y_{23} + y_{25} + y_{31} + y_{32} + y_{33} + y_{35}) + 100\,(y_{24} + y_{34} + y_{41} + y_{43} +$

$y_{54} + y_{64}) + 150\,y_{44} + 200\,(y_{42} + y_{45})$

Min $Z_2 = TS_{65}$

Subject to:

1. $TS_{15} - TS_{11} = 4$
2. $TS_{21} - TS_{11} \geq 1$
3. $TS_{22} - TS_{12} \geq 1$
4. $TS_{23} - TS_{13} \geq 1$
5. $TS_{24} - TS_{14} \geq 1$
6. $TS_{25} - TS_{15} \geq 1$
7. $TS_{25} - TS_{21} + y_{21} + y_{22} + y_{23} + y_{24} = 17$
8. $y_{21} \leq 3$
9. $y_{22} \leq 2$
10. $y_{23} \leq 3$

11. $y_{24} \leq 4$

12. $y_{25} \leq 2$

13. $TS_{31} - TS_{11} \geq 1$

14. $TS_{32} - TS_{12} \geq 1$

15. $TS_{33} - TS_{13} \geq 1$

16. $TS_{34} - TS_{14} \geq 1$

17. $TS_{35} - TS_{15} \geq 1$

18. $TS_{35} - TS_{31} + y_{31} + y_{32} + y_{33} + y_{34} = 11$

19. $y_{31} \leq 1$

20. $y_{32} \leq 2$

21. $y_{33} \leq 1$

22. $y_{34} \leq 2$

23. $y_{35} \leq 2$

24. $TS_{21} - TS_{31} + y_{31} \geq 2$

25. $TS_{22} - TS_{32} + y_{32} \geq 3$

26. $TS_{23} - TS_{33} + y_{33} \geq 2$

27. $TS_{24} - TS_{34} + y_{34} \geq 4$

28. $TS_{25} - TS_{35} + y_{35} \geq 3$

29. $TS_{35} - TS_{21} + y_{21} + y_{22} + y_{23} \leq 11$

30. $TS_{41} - TS_{21} + y_{21} \geq 4$

31. $TS_{42} - TS_{22} + y_{22} \geq 3$

32. $TS_{43} - TS_{23} + y_{23} \geq 4$

33. $TS_{44} - TS_{24} + y_{24} \geq 6$

34. $TS_{45} - TS_{25} + y_{25} \geq 3$

35. $TS_{41} - TS_{31} + y_{31} \geq 2$

36. $TS_{42} - TS_{32} + y_{32} \geq 3$

37. $TS_{43} - TS_{33} + y_{33} \geq 2$

38. $TS_{44} - TS_{34} + y_{34} \geq 4$

39. $TS_{45} - TS_{35} + y_{35} \geq 3$

40. $TS_{45} - TS_{41} + y_{41} + y_{42} + y_{43} + y_{44} = 22$

41. $y_{41} \leq 4$

42. $y_{42} \leq 4$

43. $y_{43} \leq 4$

44. $y_{44} \leq 4$

45. $y_{45} \leq 4$

46. $TS_{51} - TS_{41} + y_{41} \geq 5$

47. $TS_{52} - TS_{42} + y_{42} \geq 6$

48. $TS_{53} - TS_{43} + y_{43} \geq 5$

49. $TS_{54} - TS_{44} + y_{44} \geq 6$

50. $TS_{55} - TS_{45} + y_{45} \geq 6$

51. $TS_{55} - TS_{51} + y_{54} = 8$

52. $y_{54} \leq 1$

53. $TS_{61} - TS_{51} \geq 2$

54. $TS_{62} - TS_{52} \geq 2$

55. $TS_{63} - TS_{53} \geq 2$

56. $TS_{64} - TS_{54} + y_{54} \geq 2$

57. $TS_{65} - TS_{55} \geq 2$

58. $TS_{65} - TS_{61} + y_{64} = 7$

59. $y_{64} \leq 2.$

60. $TS_{12} - TS_{11} \geq 1$

61. $TS_{13} - TS_{12} \geq 1$

62. $TS_{14} - TS_{13} \geq 1$

63. $TS_{15} - TS_{14} \geq 1$

64. $TS_{22} - TS_{21} + y_{21} \geq 4$

65. $TS_{23} - TS_{22} + y_{22} \geq 3$

66. $TS_{24} - TS_{23} + y_{23} \geq 4$

67. $TS_{25} - TS_{24} + y_{24} \geq 6$

68. $TS_{32} - TS_{31} + y_{31} \geq 2$

69. $TS_{33} - TS_{32} + y_{32} \geq 3$

70. $TS_{34} - TS_{33} + y_{33} \geq 2$

71. $TS_{35} - TS_{34} + y_{34} \geq 4$

72. $TS_{42} - TS_{41} + y_{41} \geq 5$

73. $TS_{43} - TS_{42} + y_{42} \geq 6$

74. $TS_{44} - TS_{43} + y_{43} \geq 5$

75. $TS_{45} - TS_{44} + y_{44} \geq 6$

76. $TS_{52} - TS_{51} \geq 2$

77. $TS_{53} - TS_{52} \geq 2$

78. $TS_{54} - TS_{53} \geq 2$

79. $TS_{55} - TS_{54} + y_{54} \geq 2$

80. $TS_{62} - TS_{61} \geq 1$

81. $TS_{63} - TS_{62} \geq 1$

82. $TS_{64} - TS_{63} \geq 1$

83. $TS_{65} - TS_{64} + y_{64} \geq 4$

84. All variables are ≥ 0.

4.3 Method of Analysis

4.3.1 Solving The MOLP Model

There are two approaches for solving a multiple objectives model. The first approach solves the model a number of times with each objective in turn. The second approach takes a suitable linear combination of all objective functions and optimizes this combination. This approach requires attaching a weight or rank for each objective.

The amount of calculations involved in solving such a mathematical formulation necessitates the use of a computer. A computer is also an efficient way of organizing and storing a model. There are a number of computer software available for helping the user to input his problem in the form of a model. These programs are known as matrix generators (MGs) (Williams 1990).

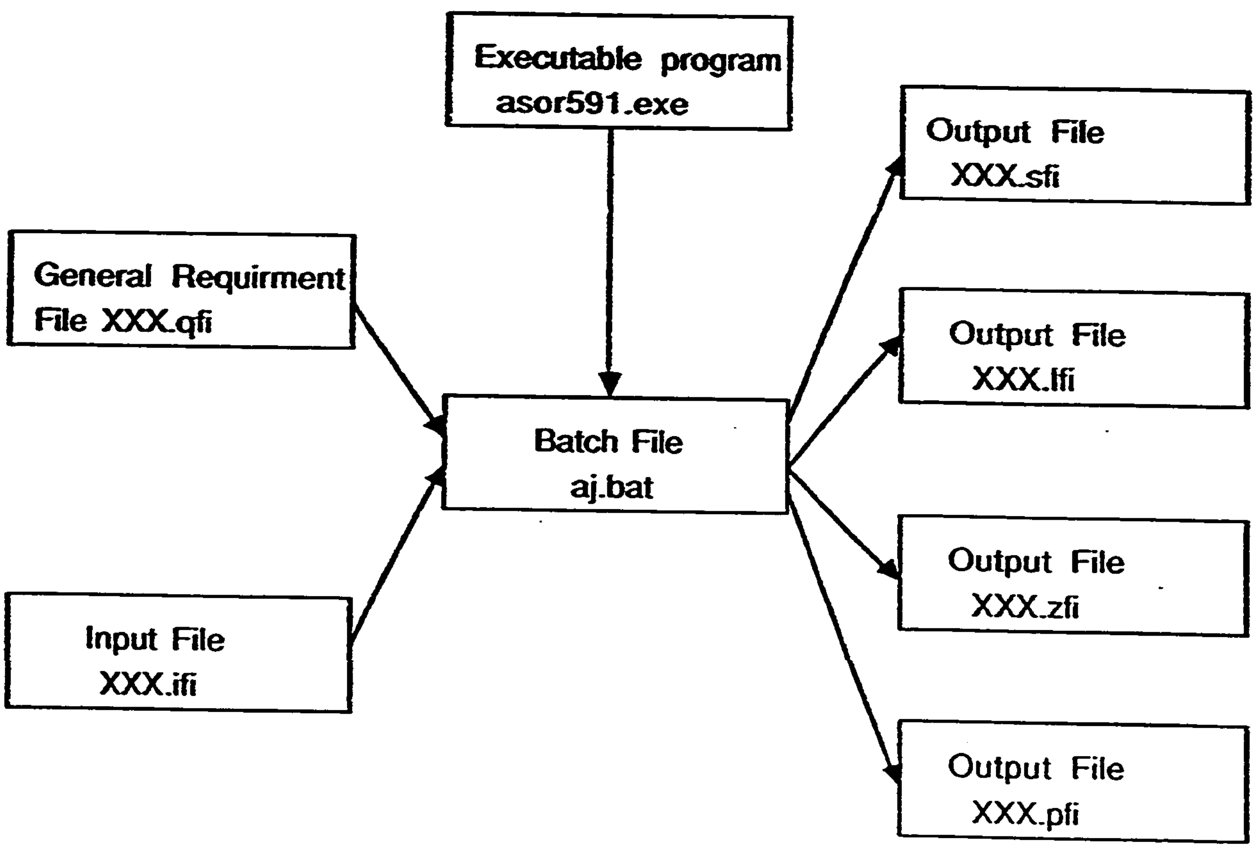
Computer output is often not clear since a package program is designed to solve any model no matter what the application is. One way to overcome this difficulty is to convert the computer output into a written report which can easily be understood. This is done through computer programs which read the solution output from the linear programming package and convert it into a clear form which relates the output to the application. These computer programs are known as report writers (Williams 1990).

An algorithm is a set of mathematical rules for solving a particular class of problem or model. There are many algorithms available. These algorithms can be programmed into a set of computer routines for solving different models. Some algorithms are coded in efficient computer programs and can be used to solve many different models. These programs usually consist of a number of algorithms collected together as a "package". Among these codes are the multicriteria simplex method presented by Zelany, the interactive approach developed by Zionts and Wallenius and ADBASE code developed by Steuer.

One advantage of these packages is that they are very flexible to use. They contain many procedures and options which may be used or ignored depending on the model requirements.

4.3.2 ADBASE Program

ADBASE is a Fortran Program for solving multiobjective linear programs for all efficient extreme points and all unbounded efficient edges. ADBASE consists of five types of files. These files are batch file, executable file, general requirements file, data input file and output file. The operation of ADBASE is portrayed in Fig. 4.14 (Steuer 1991):



ADBASE Operation (Steuer 1991)

Figure 4.14

4.3.3 Analysis

The mathematical formulation of the project example was developed first. It consists of two objective functions, 84 constraints and 47 variables. A computer program called ADBASE was used to solve the mathematical formulation because of its capability of solving MOLP problems.

The output file of ADBASE provided us with all efficient solutions for this problem. Table 4.3 shows these efficient solutions. As can be observed from Table 4.3, the project feasible duration ranges from 40 days which is the normal duration to 15 days which is the project crashed duration. The additional cost for minimization is shown with each deviation in Table 4.3. Figure 4.15 shows the relationship between project duration and the additional cost for minimizing project duration.

In the mathematical formulation of the project example, constraints 24 through 29 refer to constraints 6 and 7 in the MOLP model. Therefore, omitting these constraints will result in a formulation that does not consider time buffer constraints.

The project example was also implemented without time buffer constraints. The output under this condition provided us with different efficient solutions. Table 4.4 shows all feasible project durations versus the additional cost required to minimize the project duration.

Table No.4.3: Project Example Duration and Additional Minimization Cost

Project Duration (Days)	Additional Minimization Cost ($)
40	0
39	50
38	100
37	200
36	300
35	400
34	500
33	600
32	700
31	800
30	950
29	1100
28	1250
27	1400
26	1600
25	1800
24	2000
23	2200
22	2400
21	2600
20	2800
19	3000
18	3300
17	3650
16	4000
15	4350

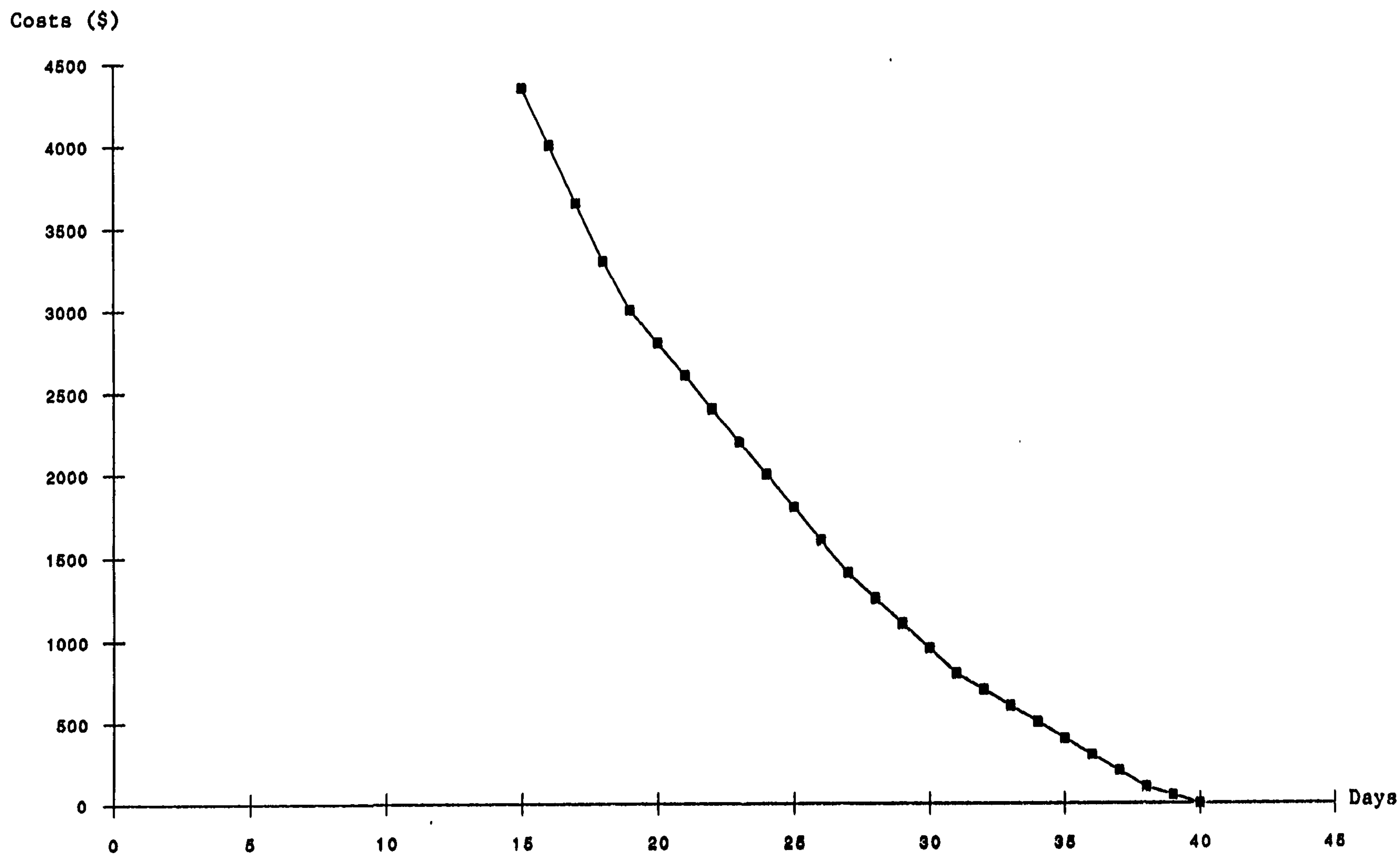


Fig. 4.15: Project Duration vs. Crashing Cost with Time Buffer Constraints.

Figure 4.16 shows the relationship between project duration and the additional cost for minimizing project duration. The project feasible durations when time buffer constraint is not considered range from 38 days to 14 days.

Table No.4.4: Project Example Duration and Additional Minimization Cost Without Buffer Constraints

Project Duration (Days)	Additional Minimization Cost ($)
38	0
37	50
36	100
35	200
34	300
33	400
32	500
31	600
30	700
29	850
28	1000
27	1150
26	1300
25	1450
24	1600
23	1800
22	2000
21	2200
20	2400
19	2600
18	2800
17	3100
16	3450
15	3800
14	4200

From the above, the effect of time buffer constraints on the results can be noticed in the project feasible duration. The project feasible duration is 2 days longer when time buffer constraints are considered.

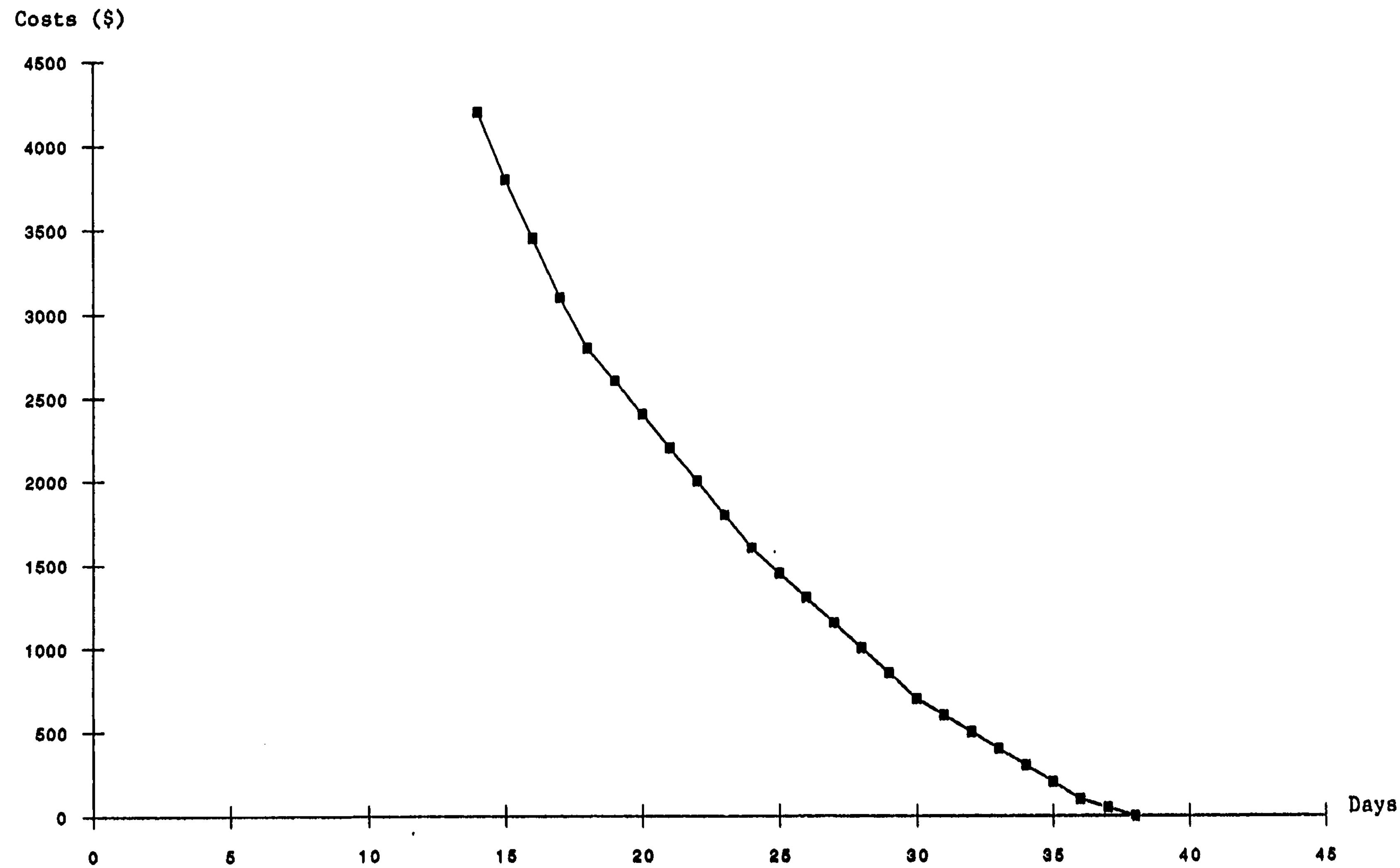


Fig. 4.16: Project Duration vs. Crashing Costs without Time Buffer Constraints.

This increase in the project duration is to meet the additional stage buffer constraints.

The output of ADBASE also provides us with the time start of each activity at each stage for each feasible solution. Table 4.5 shows the time start of the activity when the project duration is 40 days (normal duration) and the stage buffer constraints are considered.

Table No.4.5: Starting Time of Project Activities With Time Buffer Constraints

Activity	Stage				
	1	2	3	4	5
LC	0	1	2	3	4
EX	3	7	10	14	20
ST	1	3	6	8	12
LA	7	12	18	23	29
TE	27	29	31	33	35
BF	32	33	34	35	39

Table 4.6 shows the time start of project activities when the project duration is 15 days (crashed duration) and the stage buffer constraints are considered.

Table No.4.6: Starting Time of Project Activities With Time Buffer Constraints

Activity	Stage				
	1	2	3	4	5
LC	0	1	2	3	4
EX	2	3	4	6	8
ST	1	3	3	4	6
LA	3	4	6	8	10
TE	4	6	8	10	12
BF	9	10	11	12	14

Table 4.7 shows the time start of project activities when the project duration is 38 days (normal duration) and the stage buffer constraints are not considered.

Table No.4.7: Starting Time of Project Activities Without Buffer Constraints

Activity	Stage				
	1	2	3	4	5
LC	0	1	2	3	4
EX	1	5	8	12	18
ST	2	5	8	19	14
LA	5	10	16	21	27
TE	25	27	29	31	33
BF	30	31	32	33	37

Table 4.8 shows the time start of project activities when the project duration is 14 days (crashed duration) and the stage buffer constraints are not considered.

Table No.4.8: Starting Time of Project Activities Without Buffer Constraints

Activity	Stage				
	1	2	3	4	5
LC	0	1	2	3	4
EX	1	2	3	4	7
ST	1	2	3	4	7
LA	2	3	6	7	9
TE	4	6	8	10	11
BF	8	9	10	11	13

Tables 4.5 through 4.8 represent only one possible starting time of the activities. In fact, ADBASE produces all possible combinations of activities starting time for each feasible duration.

Figures 4.17 through 4.20 represent the flow line diagrams. It can be noticed that these flow line diagrams are not linear and the reason for this is that each activity has different durations at different stages in the project example. Figure 4.17 shows the flow line diagram when the project duration is 40 days (normal duration) and stage buffer constraints are considered. Figure 4.18 shows the flow line diagram when the project duration is 15 days (crashed duration) and stage buffer constraints are considered. Figure 4.19 shows the flow line diagram when the project duration is 38 days (normal duration) and stage buffer constraints are not considered. Figure 4.20 shows the flow line diagram when the project duration is 14 days (normal duration) and stage buffer constraints are not considered.

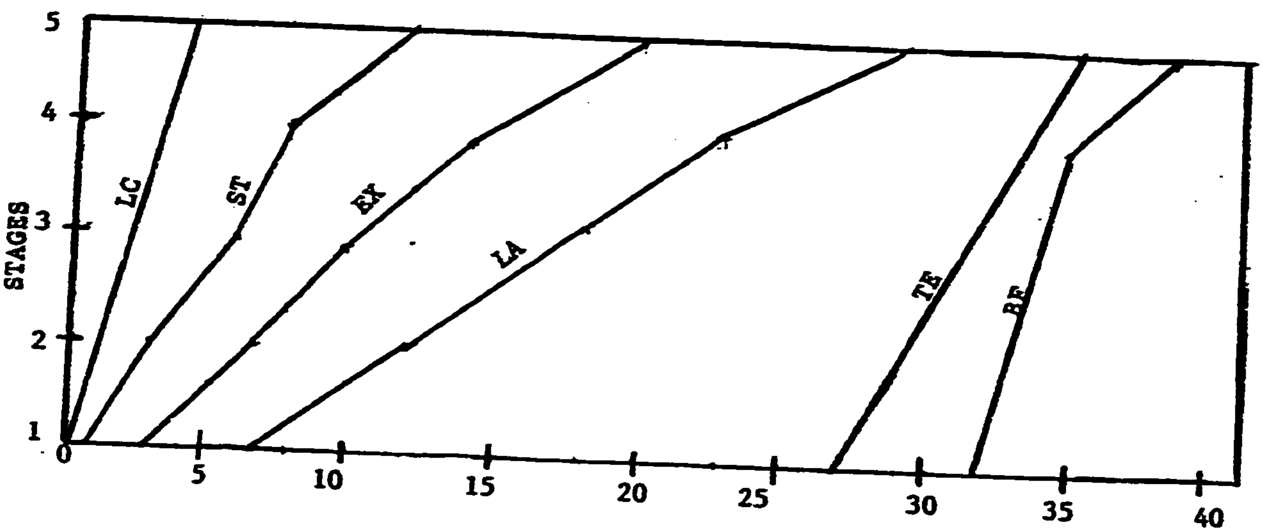


Fig. 4.17: Flow Line Curves for Project Duration of 40 Days with Stage Buffer Constraints.

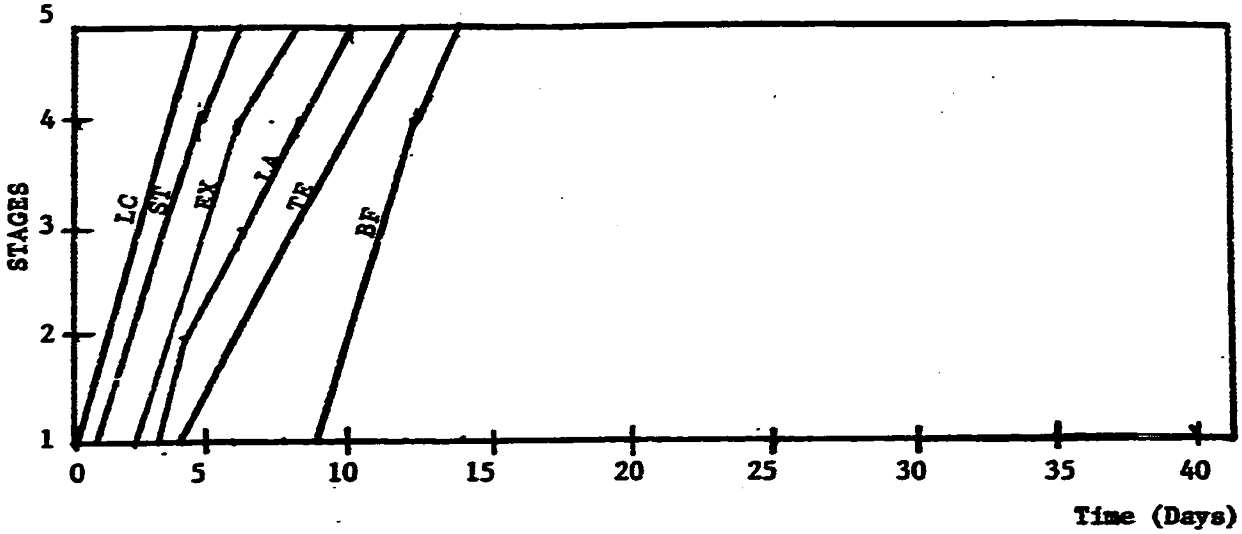


Fig. 4.18: Flow Line Curves for Project Duration of 14 days.

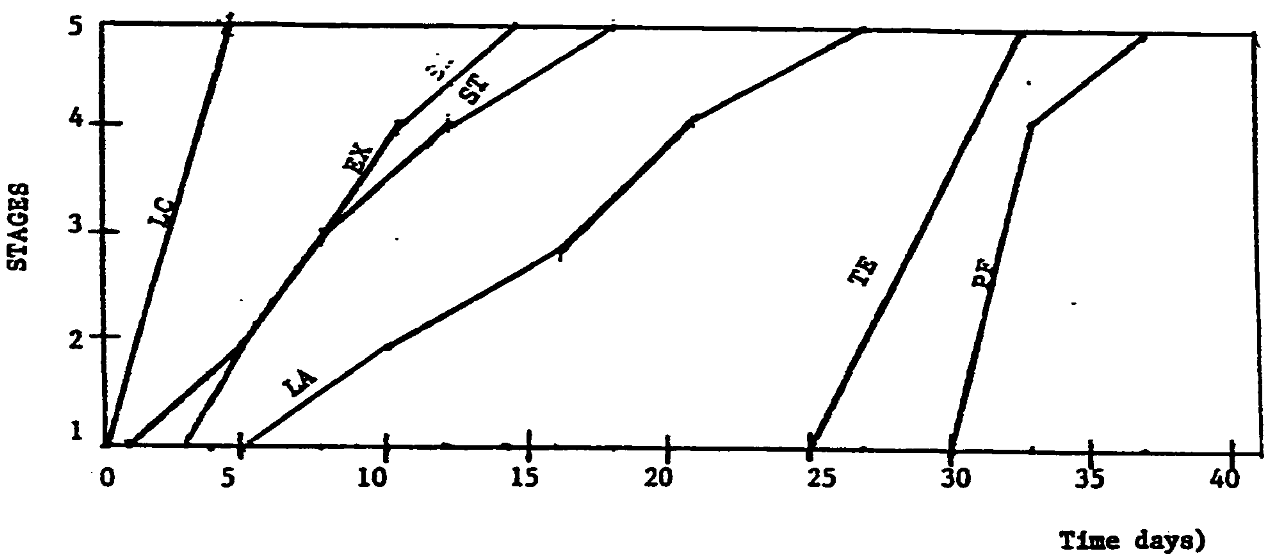


Fig.4.19: Flowline Curves for Project Duration of 38 days without Buffer Constraints.

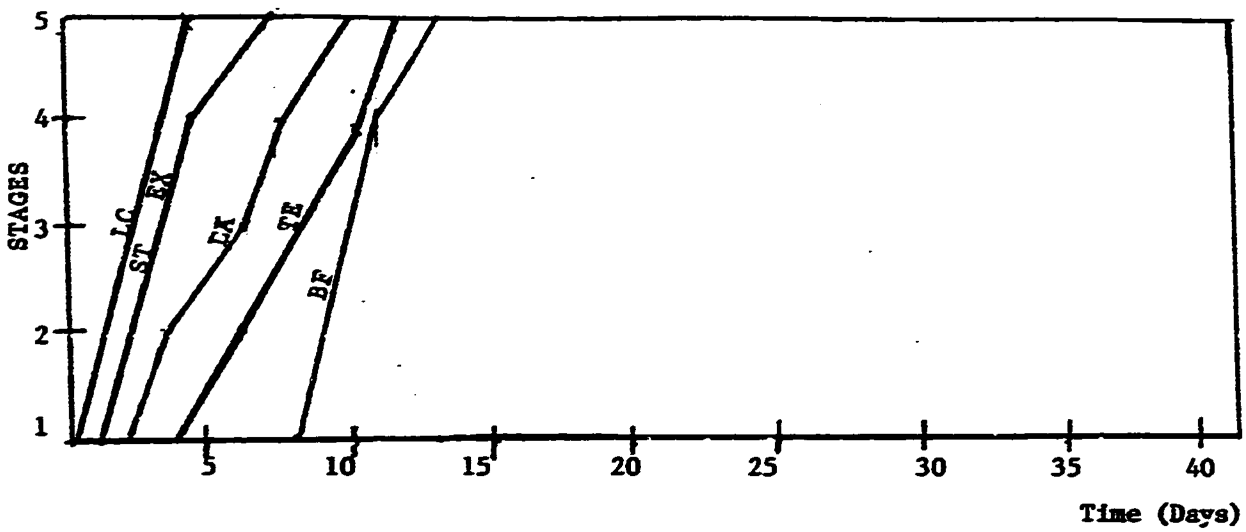


Fig. 4.20: Flowline curves for Project Duration for 14 days without Buffer Constraints.

CHAPTER V

MODEL EVALUATION

5.1 Model Evaluation

The multiple objective approach for scheduling repetitive projects provide a method for evaluating constrained problems with conflicting objectives without the necessity of specifying a weight or rank. All the objectives are evaluated simultaneously. Multiple Objective Programming develops a set of solutions, called an efficient set, rather than single solution determined by single objective models. Each member of the set is non-dominate.

The major advantages of the MOLP model are the following:

1. Minimizes both cost and duration simultaneously.
2. It assumes that an activity might have different durations at different stages.
3. It requires a simple typical stage network to represent the project and a time cost curve for each activity on the network.
4. It maintains the continuity of work.
5. The solution is represented as a set of all feasible duration and cost of the project, so the final decision will be left to the contractor.
6. It allows for stage buffer between concurrent activities.

5.2 Comparison Between MOLP and CPM

An activity on a node network for the pipeline relocation project is shown on Fig. 5.1. It represents the early start of the normal schedule for the project example with a total number of 30 activities. Figure 5.1 shows that the normal project duration is 36 days. This is compared to a project duration of 38 days as obtained through using the MOLP model. However, CPM does not maintain the continuity of work. For example, the crew working on the test activity (Activity No. 130) finishes working at the first stage on day 12 and then stays idle for 4 days until it starts working on the second stage. This idle time has a negative effect on productivity, since the learning curve effect increases with the continuity of work. The MOLP model maintains the continuity of work which gives the MOLP model a credit over the CPM method. Constraint 4 of the MOLP model maintains the continuity of work and eliminates the idle time.

The CPM schedule calculates the project duration without considering the cost involved with this duration, while the MOLP model considers both the cost and the duration.

Moreover, to shorten the project duration using the CPM can be achieved by shortening the duration of the activities along the critical path with consideration for the cost of crashing each activity. Therefore, to minimize the duration of a project using CPM requires consideration for the cost-time curve for each activity along the critical path or paths. This process of shortening the project duration is cyclic in nature and it is time consuming and tedious. On the other hand, the MOLP model produces a set of all feasible duration of the project and the cost involved in shortening the project duration. This is achieved through running the mathematical formulation of the project on a software capable of solving the MOLP problem such as ADBASE.

Figure 5.2 shows the crashed project duration which is 14 days. This crashed duration was calculated through scheduling all activities at minimum crashed duration.

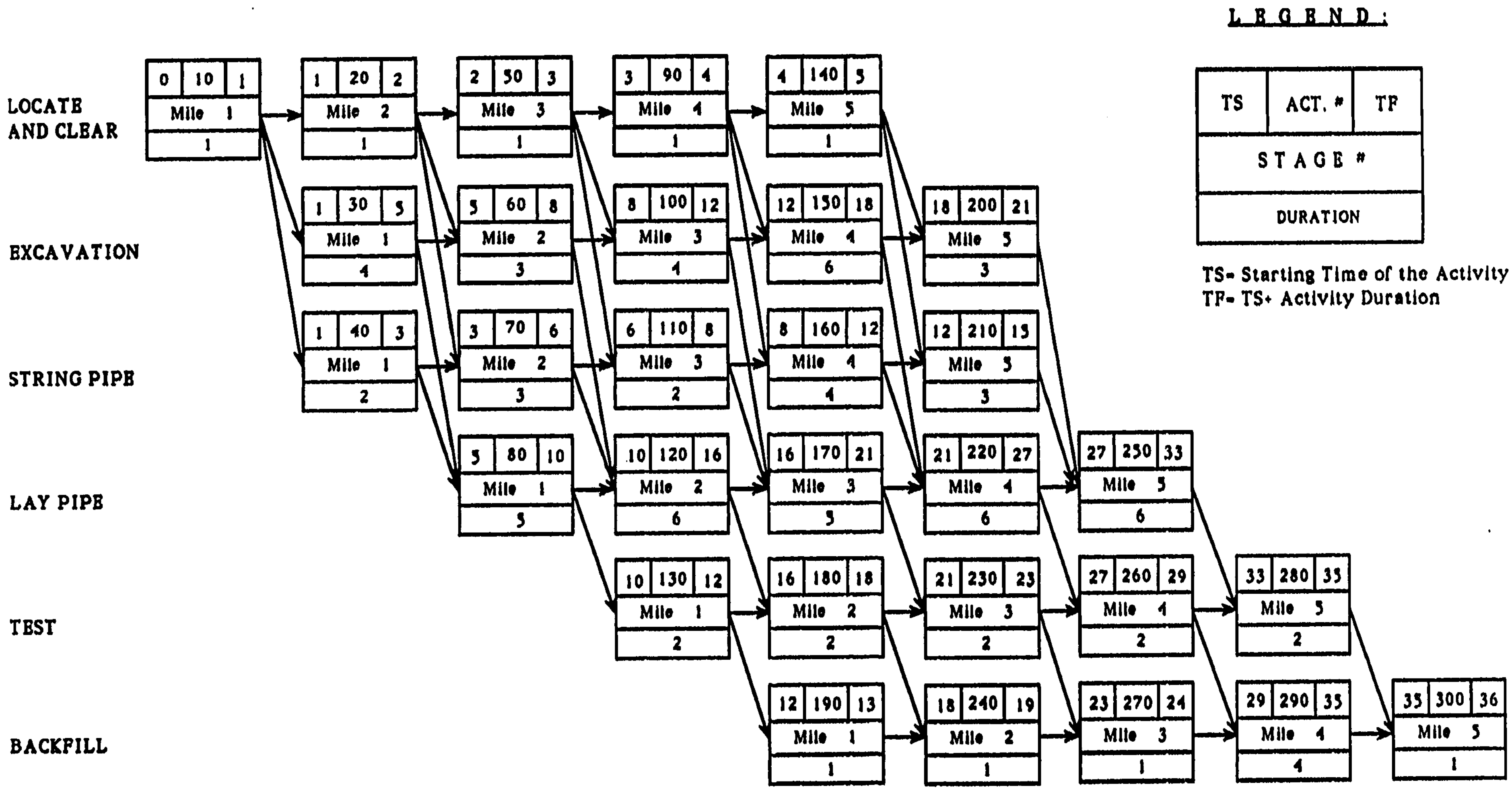


Figure 5.1: CPM Network for Example Project
Normal Duration

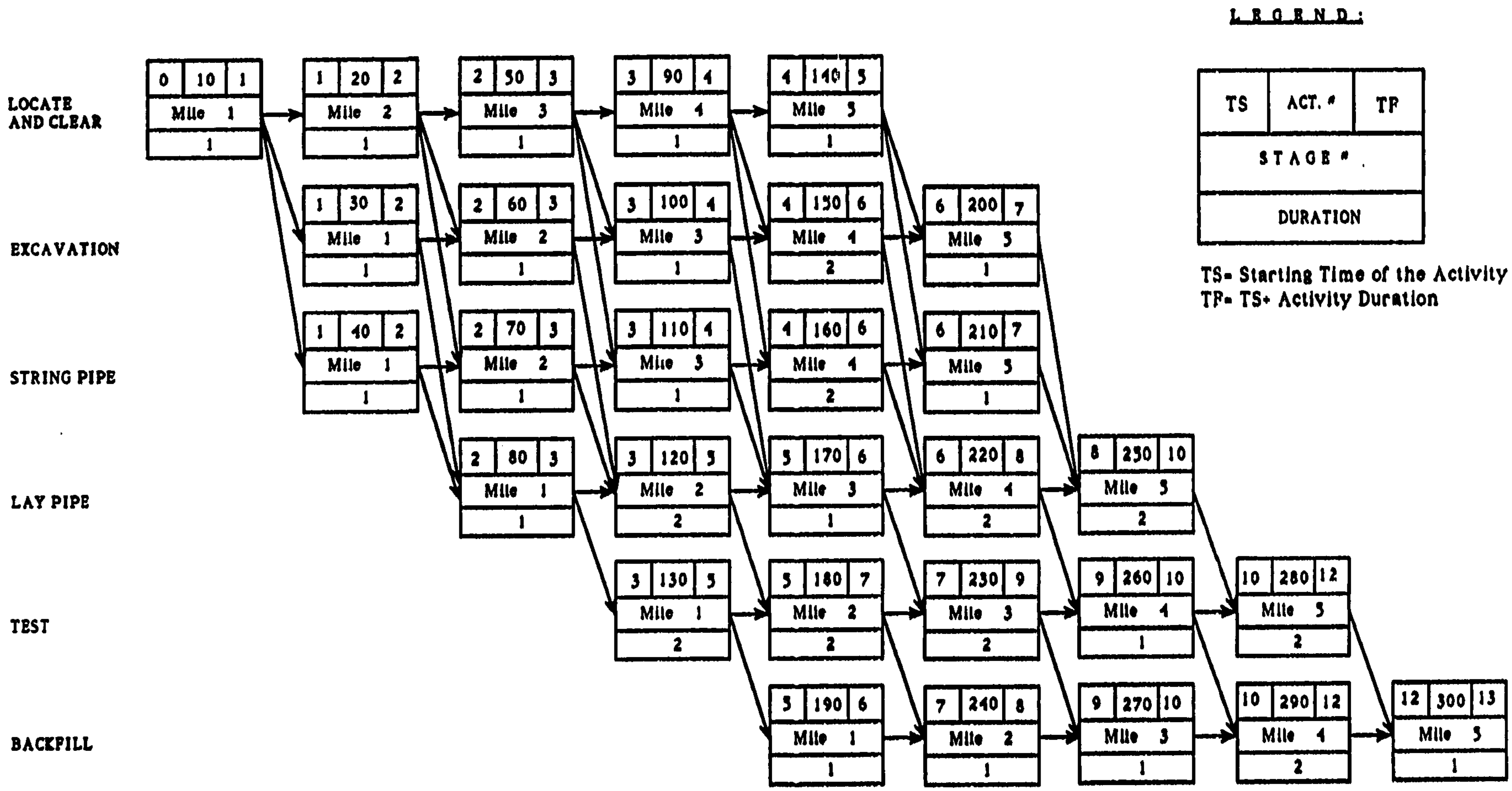


Figure 5.2: CPM Network for Example Project
Crashed Duration

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusion

This study presents a new mathematical formulation that models repetitive construction projects. The new model uses the Multiple Objective Linear Programming (MOLP) Technique. The model attempts to minimize two conflicting objectives, namely time and cost.

A review of the literature reveals that a single objective model has been developed. This model is called the Repetitive Project Model (RPM). The RPM has two drawbacks:

1. It has a single objective function for minimizing project direct cost. Since company and project indirect costs (overheads) are tied to the project duration, the model does not guarantee the minimization of the project total cost.

2. The model assumes a constant duration for each activity in all stages. This assumption might not be valid for certain repetitive projects. For example, the duration of excavation activities are usually different from one stage to another.

The MOLP model overcomes the drawbacks of the RPM. The MOLP model consists of two objective functions and 5 constraints which satisfy the following:

1. Minimize project duration.

2. Maintain constant production rate for each activity's crew.

3. Consider an activity that has different duration at different stages.

4. Allow for stage buffer between concurrent activities.

The mathematical formulation of the MOLP model has been examined using an illustrative project example. The illustrative project example was solved using ADBASE software. ADBASE software is a package which is capable of solving MOLP problems. The output of the ADBASE presents the set of all feasible duration and their costs.

The comparison between the MOLP model and the traditional CPM analysis indicates the following:

1. CPM requires to represent all activities of the project on the network even if the project is repetitive. The network for medium size construction projects is very large and it is difficult to deal with. On the other hand, the MOLP requires a network representation for one typical stage only to show the relation between the activities.
2. CPM shortens the project duration by shortening the duration of the minimum time cost slope activities along the critical path(s), while the MOLP model produces all feasible cost and duration of the project.
3. CPM does not take into account the continuity of work which has a very positive impact on the learning curve effect. On the other hand, the MOLP Model maintains the continuity of work throughout the project.

The MOLP model has several advantages over the available models of scheduling repetitive projects. Among these advantages are the following:

1. It considers both the cost and the duration of the project.

2. It assumes that an activity might have different durations at different stages.

3. It requires a simple typical stage network to represent the project and a time cost curve for each activity in each stage on the network.

4. It maintains the continuity of work.

5. It produces a set of all feasible duration and cost of the project.

6. It allows for stage buffer between concurrent activities.

7. It maintains a constant production rate for each activity.

6.2 Recommendations

The size of the MOLP model in terms of the number of constraints needed to represent a project depends on the number of activities and stages. This can be computationally inefficient for large repetitive projects, even though the computational power of most of the PC's is becoming higher and higher over time. The computational efficiency of the model needs to be studied furthermore. A package with a friendly user interface needs to be developed in order to simplify the implementations.

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